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Title: Permutation $p$-value approximation via generalized Stolarsky invariance

Abstract:  
When it is necessary to approximate a small permutation $p$-value $p$, then simulation is very costly. For linear statistics, a Gaussian approximation $\hat{p}_1$ reduces to the volume of a spherical cap. Using Stolarsky’s (1973) invariance principle from discrepancy theory, we get a formula for the mean of $(\hat{p}_1 - p)^2$ over all spherical caps. From a theorem of Brauchart and Dick (2013) we get such a formula averaging only over spherical caps of volume exactly $\hat{p}_1$. We also derive an improved estimator $\hat{p}_2$ equal to the average true $p$-value over spherical caps of size $\hat{p}_1$ containing the original data point $x_0$ on their boundary. This prevents $\hat{p}_2$ from going below $1/N$ when there are $N$ unique permutations. We get a formula for the mean of $(\hat{p}_2 - p)^2$ and find numerically that the root mean squared error of $\hat{p}_2$ is roughly proportional to $\hat{p}_2$ and much smaller than that of $\hat{p}_1$.

This is based on joint work with Kinjal Basu, Qingyuan Zhao, and Art B. Owen.