Title: Exchangeable pairs of Bernoulli random variables, Krawtchouk polynomials, and Ehrenfest Urns

Abstract:

Geoff Eagleson’s (1969) characterization of bivariate Binomial $(N,p)$ distributions of exchangeable random variables $(X,Y)$ which have Krawtchouk polynomial eigenfunctions is that the eigenvalues of the distribution have the form, for $p \geq 1/2$, as a mixture of Krawtchouk polynomials, scaled to be unity at zero,

$$\rho_k = \mathbb{E}\left[ Q_k(Z; N, p) \right],$$

for some random variable $Z$ on $\{0, 1, \ldots, N\}$.

We show that this characterization is equivalent to

$$X = \sum_{i=1}^{N} \xi_i, \quad Y = \sum_{i=1}^{N} \eta_i,$$

where $\{(\xi_i, \eta_i)\}_{i=1}^{N}$ are Bernoulli pairs with random correlation coefficients, which are conditionally independent given these coefficients.

Another equivalent pretty characterization is that the conditional distribution of $Y$ given $X$ is the same as the transition distribution in a generalized Ehrenfest Urn model.

This is joint research with Persi Diaconis, Stanford University.