A RESOLUTION OF THE TWIN PARADOX
FROM THE PRINCIPLE OF RELATIVITY

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THOMAS M. COVER and KEITH JARETT

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In the so-called twin paradox, a traveling spaceship leaves another at a velocity \( v = \beta c \), turns around after traveling some distance, and returns to the starting point. Comparing clocks at the end of the journey reveals that the stay-at-home has aged by a factor \( \gamma = (1-\beta^2)^{-1/2} \) more than the traveler. It has been argued by Dingle [1] and Sachs [2] that there should be no age difference, because either spaceship could be considered the stay-at-home. But there is a difference between the two ships—the traveler has undergone an initial acceleration and a final deceleration that could easily be detected by experimental apparatus. Nonetheless, the aging difference does not take place during these accelerations; rather the accelerations serve to place the spaceships in different reference frames for different times. (See Bondi [3].) It is the different time averages of the same in-bound and outbound relativistic doppler shifts that result in the asymmetric aging factor \( \gamma \). In our opinion, Darwin [4] has the most lucid and convincing explanation based on this argument.

In this note, we attempt to dispense with the invocation of the relativistic doppler shift factor \( \sqrt{(1-\beta)/(1+\beta)} \), which requires some beginning knowledge of special relativity, and instead derive the aging factor using only the principle of relativity. According to the principle of relativity, two unaccelerated observers receding from each other at a given uniform velocity cannot detect absolute motion. In particular, given identical clocks, each observer must measure the same doppler ratio of received clock pulse rate to transmitted clock pulse rate.

Here we must reiterate the usual assumption that, for a sufficiently
long journey, the traveler's acceleration and deceleration are unimportant, except to distinguish B from A. The principle of relativity applies to the uniform velocity segment of the travel, and the contribution to aging difference due to accelerations can be made negligibly small.

We shall use a numerical example. Traveler A stays at home. Traveler B travels at velocity $v = 3c$, $\beta = 4/5$, to a star 4 light years away. Thus A knows that B reaches the star after 5 of A's years. However, since light from the star requires 4 years to return to A, A does not see B turn around until 9 of A's years have elapsed. Traveler B actually returns to A after 10 of A's years. Finally, only light emanating from A in the first of A's years will reach B in his journey toward the star. All is revealed in the space-time diagram in Figure 1.

![Space-Time Diagram of B's Trip with A at Rest. B sees A recede for T' years. A sees B recede for 9 years. B receives A's transmissions sent during first of A's years. A receives B's transmissions sent during B's journey of T' years. Reception rates T'/9 and 1/T' must be equal.](image)
Now suppose that \( A \) and \( B \) have identical clocks emitting one pulse per year. On \( B \)'s outbound journey, we see that \( B \) receives 1 year's worth of pulses in \( T' \) of \( B \)'s years. The time \( T' \) of \( B \)'s outbound leg is still to be determined. Thus \( B \) receives pulses at rate \( R' = \frac{1}{T'} \).

The situation for \( A \) is similar. Since \( A \) sees \( B \) outbound for 9 years, \( A \) receives \( T' \) pulses in 9 years for a rate of \( R = \frac{T'}{9} \) pulses per year.

Now invoking the principle of relativity (for the uniform velocity segment of the journey), the rates \( R \) and \( R' \) must be equal. Otherwise, a preferred reference frame would be revealed. But \( R = R' \) implies \( \frac{T'}{9} = \frac{1}{T'} \), or \( T' = 3 \) years. Thus \( B \) ages 3 years on the outbound leg.

Similar calculations show that \( B \) ages three years on the inbound leg. Overall, \( A \) ages 10 years, \( B \) ages 6 years, and the age ratio is \( \frac{10}{6} = (1-\left(\frac{4}{5}\right)^2)^{-1/2} \), as predicted by the special theory.

The corresponding values are calculated in Figure 2 for arbitrary velocity \( v = \beta c \), and arbitrary distance \( x \).

![Space-Time Diagram for Velocity \( \beta c \) and Distance \( x \).](image-url)
Here $R' = ((x/\beta)-x)/T'$ and $R = T'/((x/\beta)+x)$. Finally, $R = R'$ implies $T' = x((1/\beta^2)-1)^{1/2}$. Thus the ratio of travel times is $2T/2T' = (2x/\beta)/2x (\beta^{-2} - 1)^{1/2} = \gamma$, and the desired result is obtained.

Also, since the doppler frequency ratio $\nu' / \nu$ is obviously the ratio of received clock rate to transmitted clock rate, the relativistic doppler shift is a consequence of this calculation. Thus $\nu' / \nu = R = \sqrt{(1-\beta)/(1+\beta)}$ for spaceships traveling away from each other at velocity $\nu = \beta c$.

REFERENCES


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