TIGHTENED MULTI-LEVEL CONTINUOUS SAMPLING PLANS

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GERALD J. LIEBERMAN, PROJECT DIRECTOR
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TIGHTENED MULTI-LEVEL CONTINUOUS SAMPLING PLANS

By

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1. Introduction

Industrial needs have provoked some recent studies on continuous sampling. This procedure is especially of interest when the formation of inspection lots for lot-by-lot acceptance may be impractical or artificial as in conveyor line production or there is an important need for rectifying quality of product as it is manufactured.

These newer papers are best considered in the light of the earlier papers of Dodge [3] and Wald and Wolfowitz [11]. One point of departure from the Dodge type of plan has been the introduction of several levels of partial inspection with different rates of sampling in each level. Multi-level continuous sampling plans (which reduce to the Dodge plan when only one sampling level is tolerated) have been considered by Greenwood [8], Lieberman and Solomon [9], and Resnikoff [10]. A plan based on the Wald-Wolfowitz approach, a scheme essentially handled by the methodology of sequential analysis, was created and developed by Girshick about 1948 in connection with a Census Bureau problem and has only recently been reported [7]. The reader is referred to Bowker [1] for a more thorough account of continuous sampling plans.

This paper will generalize multi-level plans of the MLP type given in [9]. The MLP plan is:

0) At the outset inspect 100 percent of the units consecutively as produced and continue such inspection until i units in succession are found clear of defects.

1) When i units in succession are found clear of defects, discontinue 100 percent inspection and inspect only a fraction f of the units (i.e., one out of every 1/f where 1/f is an integer). If the next i inspected units are non-defective, proceed to the next level; if a defective occurs, revert immediately to 100 percent inspection.

2) When at rate f, i inspected units are found clear of defects, discontinue sampling at rate f and proceed to sampling at rate f^2. If the next i inspected units are non-defective, proceed to the next level; if a defective occurs, revert immediately to sampling at rate f.

3) When at rate f^2, i inspected units are found clear of defects, discontinue sampling at rate f^2 and proceed to sampling at rate f^3. If the next i inspected units are non-defective, proceed to the next level; if a defective occurs, revert immediately to sampling at rate f^2.

... ...

k-1) When at rate f^{k-2}, i inspected units are found clear of defects, discontinue sampling at rate f^{k-2} and proceed to sampling at rate f^{k-1}. If the next i inspected units are non-defective, proceed to the next level;
if a defective occurs, revert immediately to sampling at rate \( f^{k-2} \).

k) When at rate of \( f^{k-1} \), 1 inspected units are found clear of defects, discontinue sampling at rate \( f^{k-1} \) and proceed to sampling at rate \( f^k \). If a defective occurs, revert immediately to sampling at rate \( f^{k-1} \), otherwise, continue sampling at rate \( f^k \).

During both sampling inspection and 100 percent inspection all inspected items found defective are corrected or replaced with good items.

Three generalizations of MLP are now specifically defined and are seen to be tighter plans than MLP because they require a higher rate of sampling given the same quality history for the item produced.

a) The MLP - r x 1 Plan

We say we are in the \( j^{th} \) sampling level if every \( (1/r)^j \) th item produced is systematically sampled. If \( i \) consecutively inspected items are found clear of defects when sampling at the \( j^{th} \) level, begin sampling at the \( (j + 1) \) th level. On the other hand, if a defective item is found before this is accomplished, revert immediately to the \( (j - r) \) th level, if \( j > r \), or to the zero level, that is, one hundred percent inspection if \( j \leq r \). Let inspection begin at the zero level.

b) The MLP - T Plan

This is exactly the same as the MLP - r x 1 Plan except that when a defective is encountered, we immediately revert to one hundred percent inspection. This is obviously the tightest of the three multi-level plans considered in this paper and thus bears the label MLP - T.

c) The MLP - r x s Plan

This plan follows exactly the same pattern as the MLP - r x 1 except
that when \( i \) consecutively inspected items are found non-defective while on the \( j^{th} \) sampling level, systematic sampling begins at level \((j+s)\). We shall consider the case \( r > s \) since we are concerned only with tightened multi-level plans.

2. **Summary**
   Each of these generalizations can be appraised under the assumption of an infinite number of sampling levels or a finite number, \( k \), of sampling levels. Under the assumption of an infinite number of allowable sampling levels it is possible to obtain explicit relationships between the AOQL of a plan and the parameters of the plan. Thus it is possible to graph contours of equal AOQL for each of the plans under these conditions. This makes feasible the possibility of a catalogue of continuous sampling plans which contains all the plans having a prescribed AOQL and thus aids immeasurably in the choice of an appropriate plan. As is demonstrated in the next sections the following results are obtained:

   For the MLP - \( r \times 1 \) Plan:

   \[
   \text{(2.1)} \quad \text{AOQL} = 1 - \left( \frac{r - r^{r+1}}{1 - r^{r+1}} \right)^{1/1}
   \]

   When \( r = 1 \), this reduces to the result previously obtained in [9].

   For the MLP - \( T \) Plan:

   \[
   \text{(2.2)} \quad \text{AOQL} = 1 - r^{1/1}
   \]

   This result can also be obtained heuristically by letting \( r \) approach
infinity in MLP - r x 1.

For the MLP - r x s Plan (r > s) bounds and sometimes exact AOQL's can be obtained using the previous two results. For example, if r = 4 and s = 2 and f is given, the MLP - 2 x 1 Plan for f' = f^2 will be the same plan and hence have the same AOQL.

More generally for a given f we can write

\[(2.3) \quad \text{AOQL} \quad < \text{AOQL} \quad < \text{AOQL} \quad \]

\[r'' x s \quad r x s \quad r' x s \]

where \( r' = \) greatest number less than \( r \) that is a multiple of \( s' \) and \( r'' \) is the smallest number greater than \( r \) that is a multiple of \( s \).

Under the assumption of a finite number, \( k \), of allowable sampling levels, the AOQ function for MLP - T is obtained and it is seen that the use of digital computers may be expedient for the computation of AOQL contours.

This was exactly the situation, for finite levels, in [9]. The main results of the paper are obtained through the use of Markov chain techniques which are developed in Section 3. In these plans inspection is by systematic sampling. However, the AOQ and AOQL results also hold when inspection is accomplished by random sampling.

3. Markov Chain Result

Let \( \{X_n\} \quad (n = 0, 1, \ldots) \) denote an irreducible recurrent positive Markov chain with states \( \{E_j\} \quad (j = 0, 1, \ldots) \). Let \( \{p_{ij}\} \quad (i, j = 0, 1, \ldots) \) denote the probability of transition from states \( E_i \) to \( E_j \). It is known, see [5], that a unique sequence \( \{v_i\} \) exists such that
\[ \sum_{i=0}^{\infty} v_i p_{ij} = v_j \quad (j = 0,1, \ldots) \]

(3.1) \[ v_i > 0 \quad (i = 0,1, \ldots) \]

\[ \sum_{i=0}^{\infty} v_i = 1. \]

The \( v_i \)'s are sometimes referred to as "steady state" probabilities.

Now let \( A = \{E_j\} \) be a subset of the states. Let \( Y_0, Y_1, \ldots \)
be successive members of \( \{X_n\} \) which take on values in \( A \). Since the
chain is recurrent, infinitely many such \( Y \)'s will exist with probability
one. It was shown by Derman [2] that \( \{Y_k\} (k = 0,1, \ldots) \) is also a
Markov chain; and if \( \{p'_{ij}\} (i, j \in A) \) are its transition probabilities,
then the solutions \( v'_i \) of

\[ \sum_{i \in A} v'_i p'_{ij} = v'_j \quad (j \in A) \]

(3.2) \[ v'_i > 0 \quad (i \in A) \]

\[ \sum_{i \in A} v'_i = 1 \]

are given by

(3.3) \[ v'_i = \frac{v_i}{\sum_{j \in A} v_j} \quad (i \in A). \]

Suppose \( A_1 = \{E_j\} (j = 1,2, \ldots); A_2 = \{E_j\} (j = 2,3, \ldots); \ldots \)
\( A_g = \{E_j\} (j = g, g + 1, \ldots) \ldots \) are subsets to be considered. Let
\{Y_k(g)\} denote the Markov chain defined over \(A_g\). Also let \(E_j(g)\) \((j = 0,1, \ldots )\), the states for the chain \(\{Y_k(g)\}\), be a relabeling of the states \(E_k\) \((k = g, \ldots )\) by letting \(j = k - g\). Finally let \(p_{ij}(g)\) denote the probability of transition from state \(E_i(g)\) to state \(E_j(g)\) in the chain \(\{Y_k(g)\}\). Our main tool is the following theorem.

**Theorem:**

If \(p_{ij} = p_{ij}(g)\) \((i, j = 0, \ldots ; g = 1, \ldots )\),

then

\[
(3.4) \quad v_j = v_o (1 - v_o)^j \quad (j = 1, \ldots ).
\]

**Proof:** Let \(\{v_j(g)\}\) denote the solution of \((3.1)\) for the chain \(\{Y_k(g)\}\). Since the transition probabilities, by hypothesis, are the same regardless of which chain is under consideration \(v_i(g) = v_i\) \((i = 0,1, \ldots )\). However, from \((3.3)\) we have

\[
(3.5) \quad v_o = v_o(g) = \frac{v_g}{\sum_{j=g}^{\infty} v_j} = \frac{v_g}{\sum_{j=g}^{g-1} v_j} \quad (g = 1,2, \ldots ).
\]

Thus by induction

\[
(3.6) \quad v_j = v_o (1 - v_o - \ldots - v_{j-1})
\]

\[
= v_o [1 - v_o - \sum_{i=1}^{j-1} v_o (1 - v_o)^i]
\]

\[
= v_o (1 - v_o)^j \quad (j = 1, \ldots )
\]

and the theorem is proved.
We shall apply the theorem in the following case. Suppose

\[ p_{i,i+1} = \alpha > 0 \quad (i = 0, 1, \ldots) \]
\[ p_{i,0} = 1 - \alpha \quad (i = 0, 1, \ldots, r) \]
\[ p_{i,1-r} = 1 - \alpha \quad (i > r). \]

It is clear that the chain is irreducible. It also follows from a slightly modified theorem of Foster [6] that the chain is recurrent positive if \( \alpha < \frac{r}{r+1} \). Furthermore, it is easily seen that the conditions of the theorem are satisfied so that the \( v_j \) have the form (3.4). From (3.1), \( j = 0, v_0 \) is determined by the following equation

\[ (3.7) \quad (1 - \alpha) \left\{ \frac{1 - (1 - v_0)^{r+1}}{v_0} \right\} = 1 \]

and thus any \( v_j \) can be obtained.

\[ 4. \textbf{Application to MLP - r x l Infinite Level Plan} \]

The multi-level plans can now be studied from the point of view of a Markov chain \( \{X_n\} \) and the results in Section 3 employed. We let \( E_{jm} \quad (j = 0, 1, \ldots; m = 0, \ldots, i - 1) \) denote the state of such a chain where we say that \( X_n \) is in state \( E_{jm} \) if just after the \( n^{th} \) item has been inspected the process is in the \( j^{th} \) sampling level (i.e., every \( (r^{-j})^{th} \) item inspected) and \( m \) non-defectives have been observed successively while in the \( j^{th} \) level. Suppose the process in in a state of control such that \( p \) is the probability of a defective being produced. The transition probabilities are then given by
\[ P(E_{jm} \rightarrow E_{j,m+1}) = 1 - p = q \quad (j = 0, 1, \ldots ; m = 0, 1, \ldots, l - 2) \]
\[ P(E_{j,1-l} \rightarrow E_{j+1,0}) = q \quad (j = 0, 1, \ldots) \]
\[ P(E_{jm} \rightarrow E_{j-r,0}) = p \quad (j = r, \ldots) \]
\[ P(E_{jm} \rightarrow E_{\infty}) = p \quad (j = 1, \ldots, r - 1). \]

The chain is easily seen to be irreducible. From Foster’s theorem it is seen to be recurrent positive if \( q^1 < \frac{r}{r+1} \). We shall assume \( q^1 < \frac{r}{r+1} \) for the present. Now let \( A = \{E_{jo}\} \) be a subset of the states and let \( \{Y_k\} \) denote the chain defined over it. The chain is of the form of the special case considered in section 3 with \( \alpha = q^1 \). Let \( \{v_i'\} \) and \( \{v_{jm}\} \) denote the steady state probabilities of the chains \( \{Y_k\} \) and \( \{X_k\} \), respectively. Using (3.1), (3.5) and (4.1) it follows that

\[ v_{jm} = \frac{1 - q}{1 - q^1} v'_j \cdot q^m \quad (m = 0, 1, \ldots, l - 1; j = 0, 1, \ldots) \]

For from (3.1)

\[ v_{jm} = v_{jo} q^m \quad (m = 0, \ldots, l - 1; j = 0, 1, \ldots) \]

and from (3.5)

\[ v'_j = \frac{v_{jo}}{\sum_{k=0}^{\infty} v_{ko}} \quad (j = 0, 1, \ldots) \]

Hence
\[ v_{jm} = \sum_{k=0}^{\infty} v_{ko} v'_{j} q^{m} \quad (j = 0, 1, \ldots) ; \]

but summing over \( j \) and \( m \) we get, since \( \sum_{j,m} v_{jm} = 1 \),

\[ \sum_{k=0}^{\infty} v_{ko} = \frac{1 - q}{1 - q} . \]

From (4.2) it is clear that \( v'_{j} \) is the sum of the steady state probabilities of being in the \( j \)th level of sampling. Also from (3.4)

\[ (4.3) \quad v'_{j} = v'_{o} (1 - v'_{o})^j \quad (j = 1, 2, \ldots) \]

where \( v'_{o} \) is given by (3.7) with \( \alpha = q^{\frac{1}{2}} \); namely

\[ (1 - q^{\frac{1}{2}}) \left[ \frac{1 - (1 - v'_{o})^{r+1}}{v'_{o}} \right] = 1 \]

where as previously remarked \( v'_{o} \) is the probability of being in 100 per cent inspection.

Now that we have expressions for the steady state probabilities we proceed with the derivation of the AOQ functions and the AOQL. Let

\[ h(X_n) = r^{-j} \text{ for } X_n = E_{jm} \].

It is easily verified that the reciprocal of the average fraction inspected after \( n \) inspections is

\[ (4.4) \quad F_{\frac{1}{n}} = \frac{1}{n} \sum_{v=1}^{n} h(X_v) . \]

It follows from the Birkhoff ergodic theorem, applicable for stationary
Markov chains of the type considered here (see Doob [1] p. 460) that

\[(4.5) \quad F^{-1} = \lim_{n \to \infty} \frac{F^{-1}}{n} = \sum_{j=0}^{\infty} \sum_{m=j}^{1-1} \frac{f^{-j}}{v_{jm}} \]

exists with probability one. Now $F^{-1}$ denotes the reciprocal of the average fraction inspected for all sequences (except for a set having probability 0). For let $n_k = \sum_{m=1}^{k} h(X_m) =$ number of items produced during the first $k$ inspections. Formula (4.5) says that $k/n_k \to F$ as $k \to \infty$. Let $n_k < n < n_{k+1}$. Then since $k =$ number of items inspected in the first $n$ items produced, the inequalities

$$\frac{k}{n_{k+1}} < \frac{k}{n} < \frac{k}{n_k}$$

imply that $\lim_{n \to \infty} k/n \to F$ with probability 1.

If $q^1 \geq \frac{r}{r+1}$ it can be shown more directly that $F^{-1} = \infty$ with probability 1. If $v'_o$ exists and is positive, it follows from the theory of recurrent Markov chains that $q^1 \leq \frac{r}{r+1}$. Thus since $0 < f < 1$ we have from (4.2), (4.3), (4.5) and the last remark that

\[(4.6) \quad F^{-1} = v'_o \left( \frac{1}{1 - v'_o} \right) \quad \text{when} \quad (f > 1 - v'_o)\]

= $\infty$ otherwise.

Hence since it can easily be shown that $AOQ = p(1 - F)$ we have
(4.7) \[ \text{AOQ} = (1 - q) \left( \frac{1 - f}{r} \right) \frac{l - v' o}{v' o} \quad \text{when} \quad (f > 1 - v' o) \]
\[ = 1 - q \quad \text{otherwise.} \]

Now suppose it is true that the AOQ is an increasing function of \( q \) as long as \( f > 1 - v' o \). Then from (4.7) it would follow that

(4.8) \[ \text{AOQL} = 1 - q_o \]

where \( q_o \) is the value of \( q \) such that \( f = 1 - v' o \). From (3.7) with \( \alpha = q \) it is easily established that

\[ q_o = \left( \frac{f - f^{r+1}}{1 - f^{r+1}} \right)^{1/i} \]

so that

(4.9) \[ \text{AOQL} = 1 - \left( \frac{f - f^{r+1}}{1 - f^{r+1}} \right)^{1/i} . \]

We now show that the AOQ is an increasing function of \( q \) as long as

\[ q < \left( \frac{f - f^{r+1}}{1 - f^{r+1}} \right)^{1/i} \quad \text{(i.e.,} \quad f > 1 - v' o). \]

Let \( \phi(q) = \left( \frac{f}{l - f} \right) \) \[ \text{AOQ} = (1 - q) \frac{l - v' o}{v' o} \]

and

\[ V(q) = \frac{l - v' o}{v' o} . \]
Then

\[(4.10) \quad \frac{dp(q)}{dq} = - V(q) + (1 - q) \frac{dV(q)}{dq}.
\]

It is necessary to show that the right hand side of \((4.10)\) is positive or

\[(4.11) \quad \frac{V(q)}{(1 - q) \frac{dV(q)}{dq}} \leq 1.
\]

But, using \((3.7)\) with \(\alpha = q^i\)

\[(4.12) \quad \frac{dV(q)}{dq} = \left( - \frac{1}{v'_o^2} \right) \left( \frac{1}{(1 - q)^2} \left( (r+1) (1 - v'_o)^r - \frac{1}{(1 - q)} \right) \right).
\]

Thus the left side of \((4.11)\) becomes

\[(4.13) \quad - (1 - q^i) \left[ (r+1) (1 - v'_o)^{r+1} (1 - q^i) - (1 - v'_o) \right]
\frac{i q^{i-1} (1 - q)}{i q^{i-1} (1 - q^i)}.
\]

From \((3.7)\) it follows that \((1 - v'_o)^{r+1} = \frac{(1 - v'_o) - q^i}{1 - q^i} \).

Hence \((4.13)\) becomes

\[(4.14) \quad - q^i \left( \frac{1 - q^i}{1 - q} \right) \left[ \frac{(1 - v'_o)^r}{q^i} - (r + 1) \right].
\]
But from (3.7)

\[ q^1 = (1 - v'_o) \frac{1 - (1 - v'_o)^r}{1 - (1 - v'_o)^{r+1}} \leq 1 - v'_o. \]

Hence

\[ \frac{(1 - v'_o)^r}{q^1} \geq r \]

and the smallest value over the range \( f > 1 - v'_o \) which the bracket factor in (4.14) can take is minus one. Thus the largest value that (4.14) can reach is

\[(4.15) \quad \frac{(1 - q^1)}{1 - q} \left( \frac{q^i}{i} \right). \]

But

\[ \frac{1 - q^1}{1 - q} \left( \frac{q^i}{i} \right) = q + q^2 + \cdots + q^i < 1. \]

This proves (4.11).

5. The MLP - T Plan

We consider first an infinite number of sampling levels. Let \( E_{jm} \) be as in the previous section. The transition probabilities are now

\[ P(E_{jm} \rightarrow E_{j,m+1}) = q \quad (j = 0, 1, \ldots ; 0 < m \leq i - 2) \]
\[ P(E_{j, i-1} \rightarrow E_{j+1, 0}) = q \quad (j = 0, 1, \ldots ) \]
\[ P(E_{jm} \rightarrow E_{oo}) = 1 - q \quad \text{for all} \quad j,m. \]

Of course \( 0 < q < 1. \)

It can be shown in this case that

\[ v_{jm} = pq^{j+1+m} \quad (j = 0, 1, \ldots ; m = 0, \ldots , i - 1) \]
and as before that

\[ F^{-1} = \sum_{jm} F^{-j} v_{jm} = \frac{1 - q_i}{1 - q_i f} (f < q_i^i) \]

\[ = \infty \quad (f \leq q_i^i) ; \]

and

\[ \text{AOQ} = \frac{(1 - q_i) q_i^i}{1 - q_i} \left( \frac{1 - f}{f} \right) (f > q_i^i) \]

\[ = 1 - q_i \quad (f \leq q_i^i) . \]

It can easily be shown that AOQ is an increasing function of \( q_i \) for \( 0 < q_i^i < f \). Hence

\[ \text{AOQL} = 1 - \frac{f}{i} . \]

Now let the number of sampling levels, \( k \), be finite. For this case we need only modify the function \( h(X_n) \) such that

\[ h(X_n) = f^{-j} \quad \text{when} \quad X_n = E_{jm} \quad (j \leq k) \]

\[ = f^{-k} \quad \text{when} \quad X_n = E_{jm} \quad (j > k) \]

where here we persist with the notation \( E_{jm} \) as if the \( k = \infty \) plans are in effect. In similar fashion we have
\[ F^{-1} = p \sum_{j=0}^{k-1} \sum_{m=0}^{i-1} f^{-j} q^{j+i+m} + p \sum_{j=k}^{\infty} \sum_{m=0}^{i-1} f^{-k} q^{j+i+m} \]

\[ = (1 - q^i) \frac{1 - (q^i/f)^k}{1 - q^i/f} + (q^i/f)^k \]

For \( k = 1 \), we have the Dodge Plan, and get the following result as in [3]

\[ F^{-1} = \frac{f}{f + q^i(1 - f)} \]

For \( k = 2 \)

\[ F^{-1} = 1 + q^i \left( \frac{1 - f}{f} \right) + q^{2i} \left( \frac{1 - f}{f^2} \right) \]

In order to obtain AOQL contours for this situation, as for higher values of \( k \), the use of digital computers would be expedient.
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