Stanford University
Departments of Mathematics and Statistics

PROBABILITY SEMINAR

4:15pm, Monday, March 10, 2014
Sequoia Hall Room 200
Cookies served at 3:45pm, 1st floor Lounge.

Speaker: Omer Bobrowski
Department of Mathematics,
Duke University

Title: Phase Transitions in Random Čech Complexes

Abstract:

In manifold learning, one often wishes to infer geometric and topological features of an unknown manifold embedded in a $d$-dimensional Euclidean space from a finite (random) point cloud. One topological invariant of a considerable interest is the homology of the underlying space.

A common method for recovering the homology of a manifold from a set of random samples is to cover each point with a $d$-dimensional ball and study the union of these balls. By the Nerve Lemma, this method is equivalent to study the homology of the Čech complex generated from the random point cloud.

In this talk we discuss the limiting behavior of random Čech complexes as the sample size goes to infinity and the radius of the balls goes to zero. We show that the limiting behavior exhibits multiple phase transitions at different levels, depending on the rate at which the radius of the balls goes to zero. We present the different regimes and phase transitions discovered so far, and observe the nicely ordered fashion in which homology groups of different dimensions appear and vanish. One interesting consequence of this analysis is a sufficient condition for the random Čech complex to successfully recover the homology of the original manifold.