Stanford University
Departments of Mathematics and Statistics

Probability Seminar

4:15pm, Monday, November 10, 2014
Sequoia Hall Room 200
Cookies served at 3:45pm, 1st floor Lounge.

Speaker: Jan Vondrak, IBM Almaden

Title: Exchangeability and Realizability of Random Variables on Graphs

Abstract:

De Finetti’s theorem states that every exchangeable sequence of random variables has the distribution of a mixture of i.i.d. sequences. Motivated by a recent application in graph partitioning, we study weaker notions of exchangeability and ask whether they still imply the conclusion of de Finetti’s theorem. We say that a bivariate distribution $\rho$ is $G$-realizable for a graph $G$ if there exists a joint distribution of random variables on the vertices such that the marginal distribution on each edge is the same distribution $\rho$.

Our main results are forms of de Finetti’s theorem for graphs depending on their spectral properties. We prove that if a bivariate distribution $\rho$ is $G_n$-realizable for a sequence of expanders $G_n$ with the ratio of the second and first eigenvalues tending to 0, then $\rho$ must be a mixture of i.i.d. pairs. We prove that this is also true for arbitrary orientations of expanders. These results can be viewed as extensions of results by Diaconis–Freedman and Trotter–Winkler from complete graphs to expanders.

This is joint work with T.S. Jayram at IBM Almaden.