ALLOWANCE FOR CORRELATION IN SETTING SIMULATION RUN-LENGTH
VIA RANKING-AND-SELECTION PROCEDURES

BY

EDWARD J. DUDEWICZ and NICHOLAS A. ZAINO, JR.

TECHNICAL REPORT NO. 115
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ABSTRACT

Dudewicz and Zaino (1973) investigated for the first time the effect of correlation, commonly present in simulation experiments, on ranking and selection procedures. These procedures are often the most appropriate statistical approach to the design and analysis of simulation experiments, and numerous authors (e.g., Burdick and Naylor (1966); Naylor, Wertz, and Wonnacott (1967, 1968); Kleijnen and Naylor (1969); Dudewicz and Dalal (1975); and Kleijnen (1975)) have suggested their use to set the length of computer simulation runs. Since ranking and selection procedures have generally assumed the observations are independent and since in simulation the data encountered are often correlated (e.g., within a population through lack of frequent regeneration points, and across populations through use of common random number sequences for various alternatives being simulated) those procedures may not be fully appropriate. In this paper we review the effect of correlated data on the sample size (simulation run-length) requirements of a selection procedure, detail the use of ranking and selection procedures in simulation experiments with reference to specific examples, and give methods for using ranking and selection procedures to set simulation run-length in the presence of correlation.
1. RANKING AND SELECTION IN SIMULATION

Today simulation studies play an important and ever more significant role in virtually every field of human endeavor. For example, such studies arise in business (including such areas as job-shop scheduling and marketing), the humanities (psychoanalysis), science (air pollution, chemistry, genetics), and social science (social conflict, housing policies); for detailed references to each of these applications see pp. 25-26 of Dudewicz (1975). In many cases the experimenter has under consideration several (two or more) proposed procedures for running a real-world system, and is simulating in order to determine which is the best procedure (with regard to certain specified criteria of "goodness"). In other words, often the experimenter does not wish basically to test hypotheses, or construct confidence intervals, or perform regression analyses (though these may be appropriate minor parts of his analysis): he wishes basically to select the best of several procedures (and the major part of his analysis should therefore be directed towards this goal). This is precisely the problem which ranking and selection procedures were developed for, and these procedures explicitly set sample size (in simulation this means run-length) so as to guarantee that the probability the experimenter actually selects the best procedure (called the "probability of correct selection" or P(CS)) is suitably large; for further details see Dudewicz (1976).
2. THE MODEL STUDIED

Assume given $k$ populations $\pi_1, \ldots, \pi_k$ (sources of observations which, in the context of simulation, arise from $k$ rules or procedures which are to be evaluated via a simulation experiment), the observations from $\pi_i$ denoted by $\{X_{in}\}$ where

$$X_{in} = \rho X_{i,n-1} + Z_{in} = \sum_{j=0}^{\infty} \rho^j Z_{i,n-j}$$

(1)

for some $\rho (|\rho|<1)$, and $\{Z_{in}\}$ are sequences of uncorrelated random variables with means $(1-\rho)\mu_i$ and variances $\sigma^2$. (In our notation $X_{in}$ is to be thought of as the $n^{th}$ observation from population $\pi_i$.) The problem is to select a population associated with the largest mean yield $\mu[k] = \max(\mu_1, \ldots, \mu_k)$. Effects due to having $\rho \neq 0$ (instead of the usually-assumed $\rho = 0$) were first discussed by Dudewicz and Zaino (1973) and will be summarized in Section 3, and their implications incorporated into the recommended ranking and selection procedures of Section 4.

Before discussing the assumptions behind model (1) further, we will give an example from simulation to illustrate the $X_{ij}$'s and the need for ranking and selection concepts. Lin (1975) investigated the effects of four modes of budgeting for a firm on the firm's actual profit and sales per period. The basic goal was to select that mode which yielded the highest mean value of actual profit per period (though there were subsidiary goals which required use of confidence intervals and analysis of variance procedures, the sample size was set using ranking and selection procedures, so that the basic goal of
selecting the best mode would be guaranteed with high probability.

In that context $X_{in}$ is the actual profit per period in period $n$ using
mode $i$ ($i = 1, 2, 3, 4$ since there are $k = 4$ modes of budgeting
available), and, since past decisions affect present profit, $X_{11}$,
$X_{12}$,... are not independent random variables. The problem is to
select the mode (population) with the largest mean profit per period.

For model (1), it can easily be shown (e.g., see Anderson
(1971) or Cox and Miller (1965)) that

$$E(X_{in}) = \mu_i$$

(2)

$$\text{Var}(X_{in}) = \frac{\sigma^2}{1-\rho^2} \equiv \sigma_X^2 \text{ (say)}$$

(3)

$$R_s = \text{Cov}(X_{in}, X_{i,n+s}) = \frac{\sigma^2}{1-\rho^2} \rho |s| = \sigma_X^2 \rho |s|.$$ (4)

(Note that the alternative assumptions that the observations $\{X_{in}\}$
have means $\mu_i$, common variances $\sigma_X^2$, and spacing-dependent correlations $\rho |s|$... which yield higher absolute associations between observations of closer indices... are actually equivalent to assuming the first-order autoregressive process model (1).) We assume that the $k$
populations are independent (hence there are effects on run-length of
correlation within a population, but there is no correlation across
populations), and that $Z_{in}$ is normally distributed. Since the sample
means $\bar{X}_1$ are unbiased estimators for $\mu_1$ and are asymptotically normal

---

1While the data generated by Lin were as specified, he was only interested in the 24-period (two year) performance of the new firm and therefore used 126 replicate simulation runs of 24 periods each rather than one long run of (e.g.) 126 x 24 = 3024 periods.
under general assumptions about \( Z_{in} \) (e.g., see Diananda (1953)) the results of Section 3 should be indicative for large samples even if \( Z_{in} \) is not normal.

3. THE "SAMPLE MEANS" RANKING AND SELECTION PROCEDURE'S PERFORMANCE UNDER CORRELATION

One possible procedure in the context of Section 2 (Procedure A) is to take some number \( N_3 \) of observations from each population, and choose as "best" the population which yields the largest sample mean (where "best" refers to any population with mean yield \( \mu_{[k]} = \max(\mu_1, \ldots, \mu_k) \)).

Adopting the usual indifference-zone type formulation of ranking and selection originated by Bechhofer (1954), let \((\lambda^*, P^*)\) \((0 < \lambda^* < \infty, 1/k < P^* < 1)\) be two specified constants, denote the ranked means \( \mu_1, \ldots, \mu_k \) by \( \mu_{[1]} \leq \ldots \leq \mu_{[k]} \), and try to set \( N_3 \) as the smallest sample size for which the probability of correct selection of a population associated with \( \mu_{[k]} \) (say \( P(CS) \)) is \( \geq P^* \) whenever \( \mu_{[k]} - \mu_{[k-1]} \geq \delta^* \equiv \lambda^*\sigma_X \). Then \( N_3 \) is the smallest integer satisfying

\[
\frac{1}{N_3} \left( \frac{1 + \rho}{1 - \rho} - \frac{2\rho(1 - \rho)}{N_3(1 - \rho)^2} \right) \leq \frac{1}{N} \tag{5}
\]

where \( N \) is the sample size required by the procedure of Bechhofer (1954) for the case of independent observations \((\rho = 0)\).

The fact that \( N_3 \) in Procedure A should be set as in (5) follows from the fact that our problem reduces to the framework of Bechhofer (1954) since
\[
\text{Var}(\bar{X}_1) = \frac{1}{N_3} \sum_{s=1-N_3}^{N_3-1} (1 - \frac{|s|}{N_3})_s \\
= \frac{\sigma_X^2}{N_3} \left\{ \frac{1+\rho}{1-\rho} - \frac{2\rho(1-\rho)}{N_3(1-\rho)^2} \right\} = Q(N_3) \text{(say)},
\]

so that \(X_{\bar{1}}\) being \(N(\mu_1, \sigma_X^2)\) implies \(\bar{X}_1\) is \(N(\mu_1, Q(N_3))\) and this is essentially the problem of Bechhofer (1954). With \(\rho = 0\) the necessary sample size is called \(N\) instead of \(N_3\), and the quantity \(N\) is found from the relation

\[
P^* = h_k \left( \frac{[\delta X \sqrt{N}]}{\sigma_X} \right)
\]

where \(h_k(\cdot)\) is a function tabulated by (e.g.) Bechhofer (1954) and Milton (1963). With \(\rho \neq 0\), one needs the smallest \(N_3\) such that

\(Q(N_3) \leq \sigma_X^2/N\). (Note that although \(\sigma_X^2\) is a function of \(\rho\) we delete \(\sigma_X^2(\rho)/\sigma_X^2(\rho=0)\) via \(\delta^* = \lambda^*\sigma_X\) in order to standardize our comparisons.)

For \textbf{positive correlation} \((\rho > 0)\), one can show that \(N_3\) is an increasing function of \(N\), hence we can find \(N_3\) by: solving (7) for \(N\), solving for \(N_3\) in (5) with the inequality replaced by an equality (e.g., use an iterative procedure such as the Newton-Raphson method, or the method indicated below), and rounding \(N_3\) up to the next largest integer. Alternatively one could approximate (instead of iterating) by e.g.

\[
N_2 = N \frac{1+\rho}{1-\rho},
\]

which can be shown to be conservative and (for moderate or large \(N\)) "close" to the exact solution \(N_3\).
For **negative correlation** $(\rho < 0)$ equation (5) cannot be used for non-integral $N_3$, hence a search procedure is required to find $N_3$ (see below). Approximation (8) would underestimate the necessary sample size $N_3$ (see below).

An extensive comparison of the exact and approximate solutions to (5) was made and is summarized in Table I below. For $\rho > 0$, $N_3$ was determined by solving equation (5) using IBM's Scientific Subroutine Package subroutine RTWI. For $\rho < 0$, $N_3$ was found by a search procedure starting either at $N_3 = 0$ or at $N_3 = N_1$ (here $N_1$, to be discussed in Dudewicz and Zaiño (1976), is known to satisfy $N_1 \leq N_3$) and incrementing by 1 until equation (5) was satisfied. (In all cases, $N$ was determined using the tables of Milton (1963).)

From Table I it is clear that, for a large range of the parameters involved, $N_2$ is a very adequate approximation to $N_3$.

In order to compare the sample size $N$ needed without correlation $(\rho = 0)$ with the sample size $N_3$ needed by the means procedure in the case of correlation, graphs of $M = N_3/N$ were plotted (as a function of $\Phi^*$) for: $\rho = .1, .25, .5, .6, .7, .8, .9, .95; \lambda^* = .5, 1.0; k = 2(1)5, 10, 25$. Three of these graphs ($\lambda^* = 1.0$ with $k = 25$ and $\lambda^* = .5$ with $k = 2, 25$) are given below and are indicative: for data with $0 < \rho \leq .25$ one will need (for "high" $\Phi^*$) twice the sample size as under independence, for data with $.25 < \rho \leq .8$ one will need up to eight times the sample size needed for the case $\rho = 0$, and with__

---

*For: 40 values of $\Phi^*$; $\rho = \pm .1, \pm .25, \pm .5, \pm .75, \pm .9, \pm .95; \lambda^* = .1, .25, .5, .75, 1.0, 2.0; k = 2(1)5, 10, 25.*
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<tr>
<th>$k = 2$</th>
<th>$\lambda^* = 0.10$</th>
<th>$\rho = -0.95$</th>
<th>$\rho = -0.50$</th>
<th>$\rho = -0.10$</th>
<th>$\rho = 0.10$</th>
<th>$\rho = 0.50$</th>
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<tr>
<td></td>
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<td>16</td>
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<td>111</td>
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<td>2</td>
<td>8</td>
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<td>19</td>
<td>19</td>
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Figure 1: $M = N_3 / N$ As A Function Of $P^*$ For $\lambda^* = 1.0$, $k = 25$
Figure 2: $M = \frac{N_3}{N}$ As A Function Of $P^*$ For $\lambda^* = .5$, $k = 2$
Figure 3: $M = N_3/N$ As A Function Of $P^*$ For $\lambda^* = .5$, $k = 25$
\( \rho > .8 \) one could need up to forty times the sample size required by 
\( \rho = 0 \). This sort of behavior is reasonable: as \( \rho > 0 \uparrow \), a fixed 
number of samples will yield "less" information about \( \mu \); in fact in 
the extreme case \( \rho = 1 \), a sample of size one has the same information 
as any sample of size \( > 1 \).

4. RECOMMENDED RANKING AND SELECTION PROCEDURES

Based on the above analysis over a large range of the 
parameters involved in any ranking and selection problem using the 
indifference-zone formulation of Bechhofer (1954), we believe that in 
cases in which simulation run-length is to be set using such proce-
dures the possibility of non-zero correlation \( \rho \) within the samples 
should merit serious consideration by the experimenter. Of course if 
\( \rho \) is known one may use Procedure A of Section 3 directly, with sample 
size equal to the smallest integer \( N_3 \) satisfying equation (5) (or, 
more simply, with sample size approximately equal to the \( N_2 \) of 
equation (8)).

If (as will usually be the case) \( \rho \) is unknown, and in addition 
different for different populations, the following heuristic
Procedure \( A(\hat{\rho}_i) \) is recommended. Use an initial sample of size \( N_0 = N \) 
(the sample size needed for Procedure A with \( \rho = 0 \)). Then calculate 
the estimated correlation coefficient for population \( \pi_i \) \( (i = 1, 2, \ldots, k) \)
by
\[
\hat{\rho}_i = \frac{\sum_{n=2}^{N_0} (X_{in} - \bar{X}_i)(X_{i,n-1} - \bar{X}_i)}{\sum_{n=1}^{N_0} (X_{in} - \bar{X}_i)^2}
\]

and (see Jenkins and Watts (1968), p. 191) form the 100(1-\(\alpha\))% confidence interval for \(\hat{\rho}_i\) from

\[
(\hat{\rho}_i - \hat{\rho}_i) \leq \frac{N-1}{N(N-2)} \left(1 - \hat{\rho}_i^2\right)^{\frac{1}{2}} t_{n-3} (1 - \alpha/2)
\]

with \(\alpha = .05\), where \(t_r(q)\) is the 100\(q\) percent point of Student's-t distribution with \(r\) degrees of freedom. If this 95% confidence interval contains \(\hat{\rho}_i = 0\), judge the sample size \(N\) as being adequate for population \(i\). If this 95% confidence interval does not contain \(\hat{\rho}_i = 0\), calculate

\[
N_{2i} = \left[ N \left(1 + \frac{\hat{\rho}_i}{1 - \hat{\rho}_i}\right) \right]
\]

and continue the run until we have \(N_{2i}\) observations from \(\pi_1\). Finally calculate \(\bar{X}_1, \ldots, \bar{X}_k\) based on all available observations and select (as being best) that population which produced the largest of \(\bar{X}_1, \ldots, \bar{X}_k\).

If (as will often be the case) the variances of \(\pi_1, \ldots, \pi_k\) are unknown and unequal, the following heuristic procedure \(\text{Procedure } A(\hat{\rho}_i, s_i^2)\) is recommended. Take an initial sample of \(N_0 = 30\) observations from each process. Calculate the number of observations which would be needed if we had \(\hat{\rho}_i = 0\) (zero correlation), namely (from results of Dudewicz and Dalal (1975) and their tables of \(h\))

\[
M_i = \max \left( N_0, \left[ \frac{s_i^2 h^2}{\delta^2 \sigma^2} \right] \right).
\]

12
If, as in Procedure A(\(\hat{\rho}_1\)), \(\rho_1\) is determined to be significantly different from zero, set the required sample size as

\[
N_{21} = M_1 \left( \frac{1 + \hat{\rho}_1}{1 - \hat{\rho}_1} \right)
\]

(13)

and continue the run. If \(\rho_1\) is determined not to be significantly different from zero, set the required sample size as \(M_1\).

These heuristic procedures are now being investigated in an attempt to put them on firm theoretical ground along the lines of Dudewicz and Dalal (1975). We believe (see Section 5 for details) they should be sufficient to preclude gross errors due to significant correlations.

5. SIMULATION RESULTS ON PROCEDURES A(\(\hat{\rho}_1\)), A(\(\hat{\rho}_1\), \(s_1^2\))

In Table II we give simulation results (based on 1000 replicate runs) on the performance of Procedure A(\(\hat{\rho}_1\)). From this table we see the following facts. When the autoregressive model is valid, Procedure A(\(\hat{\rho}_1\)) satisfactorily meets the probability requirement, since the attained P(CS) \(\hat{P}(CS)\) is close to the desired level \(P^*\). This holds whether 95% (\(\alpha = .05\)) or 90% (\(\alpha = .10\)) intervals are used in the basic procedure A(\(\hat{\rho}_1\)). However if the initial sample size \(N\) required (by the \(\rho_1 = 0\) case) is small, \(\hat{P}(CS)\) may be further from \(P^*\) than one would like; this does not seem serious, but can be avoided by modifying \(\alpha\).
In Table III simulation results on the performance of Procedure A(\(\hat{\rho}_1, s_1^2\)) are given, with \(\alpha = .10\). Again the attained \(P(\text{CS})\) is close to the desired level \(P^*\).

Further study of these procedures, from both simulation and theoretical approaches, is in progress, but at this point it seems clear they can be used in practice and will alleviate gross errors due to correlation within sequences of observations. Their behavior for nonautoregressive processes, and the behavior of modifications (such as: eliminate the test of \(\rho_1 = 0\) and always use a sample size based on either \(\hat{\rho}_1\) or on the upper end point \(\hat{\rho}_1^+\) of some appropriate confidence interval on \(\rho_1\)), are also under study.
### TABLE II

SIMULATION RESULTS ON PROCEDURE A($\hat{p}_d$), k AUTOREGRESSIVE PROCESSES, CORRELATION $\rho$, 1000 REPLICATIONS ($\mu_1 = 0$, $\mu_2 = .1$, AND IF k = 4, $\mu_3 = .1, \mu_4 = .1$)

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>k</th>
<th>$p^*$</th>
<th>$\lambda^*$</th>
<th>$\rho$</th>
<th>N</th>
<th>$\hat{p}_N$</th>
<th>$\hat{E}(N_2)$</th>
<th>$\hat{V}(N_2)$</th>
<th>$\hat{P}(CS)$</th>
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<tr>
<td>.05</td>
<td>2</td>
<td>.75</td>
<td>.1</td>
<td>.1</td>
<td>91</td>
<td>.745 (.718, .772)</td>
<td>99.0</td>
<td>435.6</td>
<td>.748 (.722, .776)</td>
</tr>
<tr>
<td>.10</td>
<td>2</td>
<td>.75</td>
<td>.1</td>
<td>.5</td>
<td>91</td>
<td>.680 (.651, .709)</td>
<td>259.7</td>
<td>3975.4</td>
<td>.757 (.730, .784)</td>
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<tr>
<td>.05</td>
<td>2</td>
<td>.9</td>
<td>.2</td>
<td>.5</td>
<td>83</td>
<td>.749 (.722, .776)</td>
<td>237.0</td>
<td>3502.7</td>
<td>.864 (.843, .885)</td>
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<td>.9</td>
<td>.2</td>
<td>.5</td>
<td>83</td>
<td>.756 (.730, .783)</td>
<td>238.1</td>
<td>3501.1</td>
<td>.875 (.855, .895)</td>
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<td>.1</td>
<td>.1</td>
<td>329</td>
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<td>384.9</td>
<td>3308.4</td>
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<td>.95</td>
<td>.2</td>
<td>.5</td>
<td>136</td>
<td>.820 (.796, .843)</td>
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<td>6216.2</td>
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<tr>
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<td>.95</td>
<td>.5</td>
<td>.5</td>
<td>22</td>
<td>.812 (.787, .836)</td>
<td>48.5</td>
<td>862.7</td>
<td>.871 (.850, .891)</td>
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<td>.95</td>
<td>.5</td>
<td>.5</td>
<td>35</td>
<td>.748 (.721, .775)</td>
<td>92.0</td>
<td>1609.9</td>
<td>.952 (.939, .965)</td>
</tr>
</tbody>
</table>

$N = \text{No. of observations required by procedure of Bechhofer (ignoring correlation)}$

$\hat{p}_N = \text{Estimate of probability of correct selection based on } N \text{ observations (point estimate and 95% confidence interval)}$

$\hat{E}(N_2) = \text{Estimate (based on 1000 simulated observations) of expected number of observations}$

$\hat{V}(N_2) = \text{Estimate of variance of number of observations}$

$\hat{P}(CS) = \text{Estimate of probability of correct selection using } N_2 \text{ observations}$
### TABLE III

Simulation results on procedure $A(r_0, s_0^2)$, $k$
Autoregressive processes, correlation $\rho$, 1000 replications, initial sample size $N_0 = 30$

<table>
<thead>
<tr>
<th>k</th>
<th>$P^*$</th>
<th>$\delta^*$</th>
<th>Variances</th>
<th>$\rho$</th>
<th>$h$</th>
<th>$\hat{P}_N$</th>
<th>$\hat{P}(CS)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>.892</td>
<td>.5</td>
<td>$\begin{align*} \sigma_{X_1}^2 &amp;= 1. \ \sigma_{X_2}^2 &amp;= 2. \end{align*}$</td>
<td>.5</td>
<td>1.8</td>
<td>.812</td>
<td>(.788, .836)</td>
</tr>
<tr>
<td>2</td>
<td>.9495</td>
<td>.2</td>
<td>$\begin{align*} \sigma_{X_1}^2 &amp;= 1. \ \sigma_{X_2}^2 &amp;= 2. \end{align*}$</td>
<td>.5</td>
<td>2.5</td>
<td>.828</td>
<td>(.805, .851)</td>
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<tr>
<td>2</td>
<td>.892</td>
<td>.5</td>
<td>$\begin{align*} \sigma_{X_1}^2 &amp;= 2. \ \sigma_{X_2}^2 &amp;= 1. \end{align*}$</td>
<td>.75</td>
<td>1.8</td>
<td>.709</td>
<td>(.682, .738)</td>
</tr>
<tr>
<td>4</td>
<td>.9474</td>
<td>.5</td>
<td>$\begin{align*} \sigma_{X_1}^2 = \sigma_{X_2}^2 &amp;= 1. \ \sigma_{X_3}^2 = \sigma_{X_4}^2 &amp;= 2. \end{align*}$</td>
<td>.5</td>
<td>3.0</td>
<td>.732</td>
<td>(.705, .759)</td>
</tr>
<tr>
<td>4</td>
<td>.8952</td>
<td>.5</td>
<td>$\begin{align*} \sigma_{X_1}^2 = \sigma_{X_2}^2 &amp;= 2. \ \sigma_{X_3}^2 = \sigma_{X_4}^2 &amp;= 1. \end{align*}$</td>
<td>.5</td>
<td>2.5</td>
<td>.687</td>
<td>(.658, .716)</td>
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<tr>
<td>2</td>
<td>.892</td>
<td>.2</td>
<td>$\begin{align*} \sigma_{X_1}^2 &amp;= 3. \ \sigma_{X_2}^2 &amp;= 1. \end{align*}$</td>
<td>.2</td>
<td>1.8</td>
<td>.843</td>
<td>(.820, .865)</td>
</tr>
</tbody>
</table>

$\hat{P}_N$ is estimate of probability of correct selection using procedure of Dudewicz and Dalal (ignoring correlation)
REFERENCES


