MULTIVARIATE STATISTICAL ANALYSIS AS APPLIED TO THE PROBLEMS OF UNCOVERING VOLCANIC STRUCTURES AND THE EVOLUTION OF RIFT-ZONE VOLCANISM

BY

EDWARD I. BLOOMSTEIN

TECHNICAL REPORT NO. 117
OCTOBER 6, 1976

PREPARED UNDER THE AUSPICES OF
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I. Multivariate Statistical Analysis As Applied To The Problem Of Uncovering Volcanic Structures.

Edward I. Bloomstein

Trend-analysis is used in geological applications for determination of the regularities of changes of considered variables in the space. The determination of the fitting equation with required properties is done by means of regressional analysis. The regression equation obtained is normally used while solving two sorts of geological problems:

1. The interpolation of values between fixed sample points of testing.
2. The separation of variability into a real trend and a local component.

At first the polynomial models of trend analyses are used as well as models based on the Fourier decomposition. The condition for finding the proper solution here is absence of autocorrelation in residuals between initial data and values computed on the trend surface.

Second we will be considering in detail the search for statistical solutions using two jointly displayed factors – regional and local – while generating any geological object. We would like to notice that the description of interaction of these factors is complicated by the existence of statistical noise as well as the errors in the technique of sampling of the data and their measurement.

The review of existing solutions of this problem (Merriam and Cocke, 1967; Miesh and Connor, 1968) shows that most investigators implied a trend component for the regional component, determined by either methods of trend analysis with a preliminary given class of functions or sliding fitting or their combinations.
In some cases this determination comes out to be incorrect, because usually a geologist does not know the theoretical model of behavior of the variables in the space he is considering.

A more admissible approach to evaluation of the regional component can lie on the one hand with the statistical properties of separate functions of coordinates, which compound the trend model and, on the other hand, with the properties of the surface equation entirely. The model in this case should be chosen on the class of functions appropriate for a given geological problem. As we know the general linear model of trend analysis (Krumbein, 1967) is defined by the following regression equation:

\[ Z(x,y) = \hat{b}_0 + \sum \hat{b}_i f_i(x,y), \]

where \( Z(x,y) \) - the function of coordinates, describing the trend surface \( f_i(x,y) \) - a class of functions of coordinates, defining the model; \( \hat{b}_i \) - estimations of regression coefficients, which usually are determined by the method of least squares.

The characteristic, which distinguishes our method, is the inclusion \( \{ f_i(x,y) \} \) into the surface equation only the functions for which the corresponding estimates of \( b_i \) regression coefficients significantly differ from zero. The exclusion and inclusion of functions \( f_i(x,y) \) into the regression equation is done by a stepwise regression procedure (Miesch, Connor, 1968, Acketberg, 1974) when just one function is excluded or included on each step.

To verify the hypothesis \( H_0: b_i = 0 \) at the \( k \)-th step with given significance level \( \alpha \) we use the fact that the value \( (b_{i,k}^2) / \sigma_{b_i}^2 \) (where the denominator is an estimate of dispersion \( b_{i,k}^2 \) ) is distributed as \( F_{1,m} \). In the
last expression \( m \) corresponds to degrees of freedom, which is equal to the
difference between the number of map control points of sampling and the number
of functions \( f_1(x,y) \) included in the regression equation at the \( k \)-th step.
The introduction on the \( k \)-th step of some function \( f_1(x,y) \) leads to the re-
evaluation of all regression coefficients. If they appear to be close to zero,
the hypothesis \( H_0 \) is accepted and these functions are excluded from the
equation.

The advantage of this method consists of an exclusion from the equation
of the trend surface of functions \( f_1(x,y) \) which do not carry a statistically
significant contribution to the variability of explored variables. This leads
to the decreasing of the number of terms in the equations, and to the minimum of
distortions, brought in by the local component in the process of evaluation
of the trend surface. Regression coefficients become more stable and less
dependent on the properties of samples or initial errors.

For several geological problems there is considerable interest represented
by the detection of a local component along with the regional component and
construction of corresponding maps. In this case residual anomalies are
assumed to be characterized by essential deviations of initial data from the
regional component (from the trend surface) computed at points of sampling.
The deviations from the trend surface depend on both the total influence of
locally appearing geological factors and the "noise", the value of auto-
correlation in deviations and their relative contributions. Presumably, the
scale on which the local geological factor is displayed is bigger than the
distance between neighbor map control points.
The technique of map construction submitted in this article consists of
the following stages of calculations:

1. Preparation of a regression surface on the irregularly defined values of
the initial variable from which the residuals of trend are calculated.

2. Computation of values of the fitted regression surface on the nodes of a
rectangular net (i.e., construction of a deviation map "model"). The number
of nodes is defined by the required precision of map construction and relative
sizes of areas of manifestation of the local component.

3. The correlation of the map "model" on the basis of new deviations. For
each nodal point of the model, deviations in the range of a rectangular "palett"
centered at this nodal point are considered. The size of "palett" is proportional
to the size of the whole studied surface and essentially depends on the number
of control points nearest the node contained in the "palett". If the number
of these points is less than some given quantity (in our case - five), the
size of "palett" proportionally increases. Using the deviations selected this
way the regression surface of lower order is computed and the values of the
nodal points of the "model" are changed correspondingly. The change of the
"palett" size ("pulsation") entitles us to make a map model construction even
having considerable inhomogeneity of the distribution of the assay map control
points on the plot.

4. The construction of the map of the local component on the basis of corrected
nodal points. From the author's point of view this technique of distinguishing
a regional component and constructing the maps of a local component is suitable
for many geological applications. Perhaps it will prove to be the most effective
in solving volcano-structural tasks, one of which is examined below.
We know from the theory of Williams-Anderson-Roberts [Williams, 1941; Anderson, 1936; Roberts, 1970] that negative volcano-structures (caldera-type) and positive volcano-structures (cupole-type) reflect the volume of shallow magmatic chambers. The horizontal projection (side view) of magmatic chambers which have concave or spherical forms have a rough-concentric or oval configuration on the land surface. Consequently we are able to detect and to map the ancient volcano-structures in the present topography. In volcanic areas such as the Cascades, Kamchatka, Northern Honshu, Northern Sikhote-Alin, the formation of a relief is closely connected with the geological structure due to the accumulative activity of volcanoes. The coincidence of geological and topographical surfaces for the lava plateau, volcanic ridges and separate volcanoes allows one to quantitatively analyze the distribution of relief altitudes for direct geological interpretation. Using our non-polynomial method of trend-analysis we have, first, the opportunity to determine two components in the hypsometric field and their maps, and, second, we give a volcanological interpretation to this mathematical model of structure of the territory.

Such work has been done for part of the Sikhote-Alin volcanic belt (Eastern Asia, Pacific margin of Russia, Khabazovsk region). The area of size 40 x 30 km on the near-estuary region of the Amur-river was selected for investigation (fig. 1) The relief of this area is a high partitioned plain with steep and gentle topographic elevations of isometric and impore form which is surrounded by a high ridge with conic and cupola-like summits.

For the non-polynomial trend analysis the author used 1016 coordinates of relief altitudes which were taken from the irregular net of the topographic map of this territory in scale 1:100 000. A map of the trend surface was
calculated (fig. 2) as well as the map of residuals from the trend-surface (fig. 3). From a consideration of the first map it follows that the main feature of the examined area is the north-western orientation of topographic elements which reflects the tectonic movements in this direction. Increments of altitudes towards North-east are developing simultaneously. This regular tendency consists of 55.0 percent of the total sum of squares (general variability of topographic altitudes).

A more local peculiarity of the relief in comparison with those in the first map becomes apparent under consideration of residuals from the trend surface. In the map (fig. 3) one can see a distinct ring structure which is composed of a "ridge" of positive residuals upon a "depression" of negative residuals with different amplitude.

The consequent geological mapping (Bloomstein, 1974) shows that andesite lavas, small stocks and plugs are distributed everywhere in this territory. Dacit plugs and lavas have a subordinate role and were regarded as an acid differentiate of andesite magme. All these volcanics were subdivided into two series: 1. High-potassic andesites and dacites of senonian age (K-Ar determination 75-80 m.y.) 2. High-alumina basaltic andesites, andesites and dacites of Miocene age (K-Ar determination 12-16 million years) The succession of eruptions was established and a large number of ring faults and linear zones of fractures with north-western, meridional and latitudinal extension were traced. It was determined that some of the above mentioned landforms represent andesite and rhyolite eruptive centers and separate larger scale volcanoes. The geological information as well as the character of residuals from the trend surface permit one to interpret this annular structure as a volcano-tectonic one,
composed of a calderic depression and ridge generated by post-caldere volcanoes
sitting on the arcuate magma-transport fault. The uncovered volcano-structure
received the name New Amur; its diameter was 35 km. The amplitude of subsidence
was calculated to be 200m, using the difference in the position of the roof of
senoniah andesitic lava rock in the bottom of the caldera and in the over-
caldera ridge.

The small amplitude of subsidence, the moderate thickness of the volcanics
and the considerable diameter of the shallow caldera might testify to a lopolithic
or sub-layer form of the magmatic chamber.

Another important feature of the map of residuals is the possibility of
objective detection of fracture zones. They may be determined in two ways:
1) by using residuals of one sign, which generate linear elongated zones on
the background of residuals of the other sign, 2) by using the steepest
gradients of residuals of one sign in certain sections of the area. The
meridional, north-east and north-west faults determined by this method are
shown in figure 3. As far as modern topographic maps made on a very precise
stereophotogrammetric basis are concerned, there is nothing surprising that
these faults coincide well with faults determined by geological mapping.

Thus from the analyzed example one can see that if we have a geological
prerequisite, non-polynomial trend analysis is a useful method of detection of
ancient volcano-tectonic structures in those regions of development of a
Cretaceous and Neogene volcanism, which are distinguished by small and medium
levels of erosion. In such regions volcano-structures or their ruins as well
as zones of fractures usually are clearly expressed in modern topography.

The empirical nature of trend-analysis and the researcher's lack of a
theoretical justification of the validity of a choice of a concrete mathematical
model allow one to choose such a partition of the regional and local component which in the best way corresponds to the geological interpretation and characterizes the high fidelity of fitting of the regression surface.

The algorithm of stepwise regression was programmed in FORTRAN for MINSK-32 computed by Victor I. Mishin of the Geological Research Institute (Leningrad, USSR), the criteria of probability level was adopted from A. Miech and J. Connor.

The critical reading and comments by Dr. Paul Switzer are very much appreciated. The assistance of N. Stepanov is gratefully acknowledged.
References


Fig. 1. Map of the Trend Surface
Fig. 2 Factual Map of Residuals from the Trend Surface
Fig. 3  Generalized Map of Residuals. Large dots denote Area of Negative Residuals below -150. Small dots denote Area of Positive Residuals Above +150 and 300.
II. Evolution of Rift-zone Volcanism

Preface

The development of rift zones is accompanied by strong manifestations of volcanism. Rift genesis and magma genesis are both conditioned by processes taking place in the upper mantle. New data from the ocean floor permit us to carry out a comparative analysis of the volcanism in continental and oceanic rift zones with the aim of determining similarities or differences in their development.

Input Data

The present author and Dr. Andrew Grachev (Mishin, Grachev, Bloomstein, 1974) have collected and prepared for processing 46 sets of petrochemical data (1500 complete chemical analyses in all), which characterize the volcanism of most well-known late Cenozoic continental and oceanic rift zones, as far as their more ancient analogs (table 1). The selection of analyses was made according to a geological-structural principle which makes the inclusion of non-rift rocks into the samples improbable. Exceptions were made for trapp basalts of the Siberian platform and for spilitic basalts of the Southern Ural Mts. because it has been stated that mid-oceanic ridge volcanism is identical to initial geosynclinal volcanism. The treatment of input data to obtain a classification of basaltic series were carried out by means of cluster and R-mode factor analysis.

Cluster analysis

The task of classifying the studied objects when a priori information
List of Investigated Basalt Samples (n=1532)

<table>
<thead>
<tr>
<th>Sample number</th>
<th>Name</th>
<th>Number of analyses</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>CONTINENTAL RIFT ZONES</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Baikal rift zone</strong> (East Siberia, USSR)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Chara rift</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Middle Quaternary Basalts</td>
<td>60</td>
</tr>
<tr>
<td>24</td>
<td>Upper Quaternary-Holocene Basalts</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td><strong>Tunka rift</strong></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Miocene Basalts of the Tunka Depression</td>
<td>27</td>
</tr>
<tr>
<td>4</td>
<td>Pliocene Quaternary Basalts of Tunka Depression</td>
<td>28</td>
</tr>
<tr>
<td>11</td>
<td>Miocene Basalt of the Khamar-Daban Mountains</td>
<td>21</td>
</tr>
<tr>
<td>16</td>
<td>Upper Quaternary Holocene Basalts of the Dzida River</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Vitim Plateau</strong></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>Quaternary Basalts</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td><strong>Mona rift zone</strong> (North-East Asia, Kolyma region, USSR)</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>Basalts of Balagan-Tas Volcano</td>
<td>18</td>
</tr>
<tr>
<td>18</td>
<td>Basalts of Amui Volcanoes</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td><strong>East African rift zone</strong></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Trapp Series of Ethiopian Rift</td>
<td>22</td>
</tr>
<tr>
<td>20</td>
<td>Quaternary Basalts of Ethiopian Rift</td>
<td>13</td>
</tr>
<tr>
<td>17</td>
<td>Basalts of Afar Triangle</td>
<td>28</td>
</tr>
<tr>
<td>19</td>
<td>Basalts of Aden Series</td>
<td>14</td>
</tr>
<tr>
<td>1</td>
<td>Basalts of Tana Lake</td>
<td>14</td>
</tr>
<tr>
<td>7</td>
<td>Neogene-Quaternary Basalts of North East Syria</td>
<td>32</td>
</tr>
<tr>
<td>28</td>
<td>Floor Basalts of the Red Sea</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td><strong>Western United States rift zone</strong></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Quaternary Basalt of Mojave Desert</td>
<td>15</td>
</tr>
<tr>
<td>10</td>
<td>Miocene Basalts of Northern Oregon</td>
<td>21</td>
</tr>
<tr>
<td>8</td>
<td>Pliocene-Quaternary Basalts of the Snake River</td>
<td>32</td>
</tr>
<tr>
<td>9</td>
<td>Basalts of Rio Grande Rift</td>
<td>17</td>
</tr>
<tr>
<td>36</td>
<td>Basalts of New Mexico</td>
<td>17</td>
</tr>
</tbody>
</table>
OCEANIC RIFT ZONES

Atlantic Ocean

Floor Basalts in area 15-20° n.l. 28-45° w. longitude
Quaternary Basalts of Middle Rift of Iceland

Islands of Mid-Atlantic Ridge

Tristan-da-Kunhe
Gough
St. Helena
Jan-Majen
Tenerife
Madeira

inside ridge zone
outside of ridge

Indian Ocean

Floor Basalts of Carlsberg Ridge
Reunion Island, Oceanite Series
Reunion Island, Piton-de-Nege
Reunion Differentiation Series
Mohali Island

Pacific Ocean

Eastern Pacific Uplift

Floor Basalts
Islands

ANCIENT RIFT ZONES

Mesozoic Rift of Eastern Transbaikal Region
Mesozoic Rift of Western Transbaikal Region

Pre-Cambrian Riffs

Russian Platform
Pripyat
Parandov
Pachelma

North American Platform
Basalts of Coppermine River

Siberian Platform
Low Cretaceous Basalts of Franz Joseph Land
Low Triassic Basalts of Taymyr Peninsula
Low Triassic Basalts of the North of Siberia
Spilites of Silurian in the Southern Ural Mts.
about distribution functions is absent may be aided by cluster analysis. This method permits one to group the ensemble of objects into homogeneous classes on the assumption that groups of "closely" situated points correspond to separate classes. The introduction of a monotonic measure of proximity is determined by the essence of the problem under consideration. Cluster analysis allows one to create formalized models as a basis for future investigations. The theoretical prerequisites of cluster analysis are established in especially interesting articles of G. Lance and W. Williams, [Lance, Williams, 1971]. However at this point in time, a strict theory of cluster analysis does not yet exist which considers the difficulties in the choice of the model of the studied system, as well as in the interpretation of results. These results, to a considerable degree depend on the following: the accepted measure of the similarity among objects, the algorithm of classification employed, and the criteria of significance for distinguishing between classes [Van Ness and Fisher, 1971; Parks, 1966; Rao, 1971; Rubin and Friedman, 1968; Scott and Simmons, 1971].

None the less, cluster analysis is finding wide acceptance in various fields of earth science [Marriott, 1971; Sneath and Sokel, 1973; Symons and Ringele, 1976; Jaquet, Froidenvaux and Vernet, 1975]. We have previously examined the existing various techniques of clustering and found that the paragroup method [Dudenko, 1972] is the best way to classify basaltic series.

The paragroup method and principal component method.

Here we consider the problem of classifying the $K$ samples. The $i$-th sample (see table 1) is represented by $n_i$ replications (chemical analyses)
of a vector $X = (x_1, x_2, \ldots, x^m)$. The components are weight percentages of the following oxides:

We assume the hypothesis of equality of covariance matrices:

$$H_0: \ W_1 = W_2 = \cdots = W_K.$$  

The weighted estimate of the covariance matrix is

$$\hat{\Sigma} = \frac{1}{N-K} \sum_{i=1}^{N} (n_i - 1)W_i, \quad N = \sum n_i.$$  

The square root of the generalized Mahalanobis distance is a reasonable quantity with which to represent the measure of similarity of objects. The distance between sample A and sample B is determined by the formula

$$D_{AB}^2 = (M_A - M_B)W^{-1}(M_A - M_B)$$

where $M_A$ and $M_B$ are the sample mean vectors. As high correlation coefficient between variables in the initial petrochemical system lead to a special or badly conditioned covariance matrix, the use of the principal components method of factor analysis allows us to avoid such difficulties. By this method we reduce the number of variables and finally to go on to statistically uncorrelated variables (principal components) with only a small loss of information.

The correlation matrix common for all samples can be evaluated by formula:

$$R = D_W^{-1/2}WD_W^{1/2}$$

where $D_W$ is a diagonal matrix composed of the diagonal elements of the
matrix W. The initial data transformation by the principal components method means the following:

\[ D = \Lambda^{1/2} U^T X_0 \]

where \( \Lambda \) is a diagonal matrix composed of eigenvalues of the matrix \( R \);
\( U \) is the matrix of eigenvectors and \( X_0 \) is a matrix equal to \( XD_W^{-1/2} \);
\( F \) is a vector of \( m \) statistically uncorrelated variables (principal components).

After this transformation we use as new variables \( m' \) principal components \((m' < m)\) depicting the main part of the variability of the initial system (usually around 20%). The Mahalonobis distance becomes equivalent to the square of the distance in Euclidian space. We obtain the matrix of generalized distances which can be considered as a matrix of paired intersample distances.

Now we come to use of paragroup method. [Dudenko, 1972]. The paragroup algorithm consists of combining at each stage those groups for which the minimal value of the function \( T(i,j) \) is reached:

\[ T_{i,j} = \frac{S_{ii}^W + S_{jj}^W + S_{ij}^B}{(n_i + n_j)(n_i + n_j + 1)/2} \]

where \( S_{ii}^W \) is a quantity characterizing the internal group distance of the \( i \)-th group; \( S_{ij}^B \) is a quantity characterizing intergroup distance between \( i \)-th and \( j \)-th groups. Initially each sample is a distinct group. Groups \( K \) and \( \ell \) in which the minimum of \( T(i,j) \) is reached are unified into a group having the label \( t = \min(K, \ell) \) and the quantity
\[ S_{tt}^W = S_{kk}^W + S_{k\ell}^W + S_{k\ell}^B \]

is considered as the internal group distance of the new group \( t \). The intergroup distance is recalculated by the formula

\[ S_{t\ell}^B = \max_{i \neq k, \ell} S_{\ell i}^B. \]

The group with the number \( t' = \max(k, \ell) \) is excluded from consideration.

The process is repeated until either the selected criterion of homogeneity of formed groups indicates the uselessness of further merging or all initial objects are united into a single group. The last situation corresponds to the hierarchical systems and allows us to represent the results of the classification as a two-dimensional diagram (dendrogram) which reflects the reciprocal interrelations between groups as well as within groups. For the two-dimensional graphic expression of the general interrelation among objects, which in reality is multidimensional, the dendrogram [McCann, 1968; McCann, Wenninger, 1970] is more exact than a one-dimensional dendrogram. As a criterion of group homogeneity one usually uses an average within-group distance between pairs of objects. This permits one to correspondingly determine the criterion of intergroup similarity as an average intergroup pair distance. We should underline that the dendrogram does not permit one to compare objects in different groups, but shows only a distance between groups, to which they belong.

**Obtaining classifications and the "Bowen trend"**

The paragroup method of cluster analysis was applied to the principal
components (factor variables) calculated from the original input data. The method produced the dendrograph of figure 1.

As one can see from the dendrograph, the aggregate of studied samples is decomposed into two branches, one of which, in turn, is divided into another two branches. This partition is not accidental and has a petrological sense.

Let us consider the result of a factor analysis performed separately on each of these four groups, which enables us to detect the main trend of petrochemical variability (first principal component) and to establish the degree with which these components participate in the total variance (i.e. their "weights"). Table 2 shows that the "main trend" of the evolutionary process of basalt magmas in continental and oceanic rift zones should be interpreted as a "Bowen trend". This trend is expressed in the formation of a bi-polar first principal component, in which the correlated behavior of silica, alumina and alkalis are contrasted to the close correlated association of high-melting oxides. We believe that such a statistical performance of these oxides could arise in the course of the pre-crystallization acid-basic differentiation of elements and fractional differentiation. Established types of basaltic series differ according to: 1) whether or not they manifest the "Bowen trend"; 2) its intensity (expressed in the weight of the first principal component); 3) its degree of completeness, i.e. the quantity of oxides with correlated behavior included in the first component.

In primitive tholeiites of mid-oceanic ridges as well as in Icelandic tholeiites, the "Bowen trend" is practically absent. On the contrary, in
Fig. 1. Dendrograph of basalt rift series. On the vertical axis is within-group distance; on the horizontal axis are sample identification numbers from Table I.
<table>
<thead>
<tr>
<th>Types of basaltic series determined by cluster analysis</th>
<th>&quot;Main trend&quot; of the volcanic process (first principal component)</th>
<th>&quot;Weight&quot; of main trend (percentage of variance)</th>
<th>Interpretation of &quot;main trend&quot;</th>
<th>Basaltic suites (numbers of individual samples refer to column 2, table 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alkali type</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Properly alkaline sub-type</td>
<td>$\frac{+ Al_2O_3 + Na_2O - FeO}{MgO + CaO}$</td>
<td>32.6</td>
<td>Incomplete, weakly expressed Bowen trend</td>
<td>Samples 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 13, 14, 15, 16, 18, 32, 33, 34, 36, 41 - Alkali olivine basalts of Baikal, East African, West American and Moma rift zones, islands of Madeira, Reunion, Moheli, as well as ancient Pripiat anolocogene</td>
</tr>
<tr>
<td>Transitional type</td>
<td>$\frac{+ S_i O_2 + Al_2O_3 + Na_2O - FeO}{MgO + CaO}$</td>
<td>68.9</td>
<td>Complete, strongly expressed Bowen trend</td>
<td>Samples 21, 22, 23, 24, 25, 26, 35 - Nepheline-normative basalts-trachytes from islands of Mid-Atlantic, Mid-Indian oceanic ridges, East Pacific Rise, as well as recent basalts of Chaar rift in the Baikal rift zone</td>
</tr>
<tr>
<td>Tholeitic type</td>
<td>$\frac{+ S_i O_2 + K_2O + Na_2O - FeO}{MgO}$</td>
<td>30.1</td>
<td>Incomplete, weakly expressed Bowen trend</td>
<td>Samples 10, 12, 17, 19, 20, 38, 39, 40, 42, 43, 44, 46 - Quartz tholeitic triangle of Afar, Aden province, Jan Majen island, Central Oregon, Coppermine province, Mesozoic depressions of Western and Eastern Transbaikal region</td>
</tr>
<tr>
<td>Tholetesian</td>
<td>$\frac{+ T_i O_2 + FeO + K_2O}{MgO}$</td>
<td>27.5</td>
<td>Bowen trend not apparent</td>
<td>Samples 27, 28, 29, 30, 31, 37, 45 - Olivine-hypersthene tholeites of mid-oceanic ridges in Atlantic, Indian and Pacific Oceans, Red Sea, and Siberian platform</td>
</tr>
</tbody>
</table>
alkali olivine basalts of continental rifts (East Africa, Baikal, Western United States) the "Bowen trend" does make a significant contribution to the evolution (weight of first component 30-65%). Finally, in alkali olivine basalts in the mid-Atlantic and Pacific Islands, the "Bowen trend" is strongest (50-75%) and is the main trend of magmatic differentiation.

The average chemical composition, CIFW norms and some petrochemical parameters for the rocks of each group are presented in Table 3. On the basis of these data as well as geological information about samples, the four groups from figure 1 may be characterized grossly as follows:

1. Undifferentiated alkali olivine basalts from fissure eruptions;
2. Alkali basalts, strongly differentiated to trachytes, which erupted from central type volcanoes;
3. Quartz tholeiites;
4. Olivine-hyperstene tholeiites with low content of potassium, formed during fissure eruptions.

Evolution of rift-zone volcanism

It is possible to discuss the evolution of rift-zone volcanism in either a broad or a narrow sense. In the second case we are talking about whether both these chosen basaltic types have always existed or whether they have appeared at a certain stage of the geological history of the earth. In the broad sense we consider the variation of petro-chemistry during the process of rift development.

As one can see from the data (table 1,2,3; fig. 1), the volcanism of Precambrian rifts (i.e. auocogenes and geosynclinal troughs), Mezozoic
Table 3. Average analysis, CIPW norms and some petrochemical parameters for established basaltic series of rift zones (sum normative minerals 100%).

<table>
<thead>
<tr>
<th>Group</th>
<th>I</th>
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<th>III</th>
<th>IV</th>
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<tr>
<td>SiO2</td>
<td>46.93</td>
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<td>2.15</td>
<td>2.21</td>
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<tr>
<td>Al2O3</td>
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<td>17.35</td>
<td>13.78</td>
<td>15.63</td>
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<tr>
<td>Fe2O3</td>
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<td>3.50</td>
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<tr>
<td>FeO</td>
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<td>5.30</td>
<td>7.30</td>
<td>8.11</td>
</tr>
<tr>
<td>MgO</td>
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<td>3.49</td>
<td>6.28</td>
<td>7.89</td>
</tr>
<tr>
<td>CaO</td>
<td>8.69</td>
<td>6.38</td>
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</tr>
<tr>
<td>K2O</td>
<td>1.52</td>
<td>3.16</td>
<td>1.47</td>
<td>0.34</td>
</tr>
<tr>
<td>Q</td>
<td></td>
<td></td>
<td>1.92</td>
<td></td>
</tr>
<tr>
<td>OR</td>
<td>9.17</td>
<td>19.07</td>
<td>9.02</td>
<td>2.04</td>
</tr>
<tr>
<td>Ab</td>
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<td>33.14</td>
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<tr>
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</tr>
<tr>
<td>Il</td>
<td>4.36</td>
<td>4.17</td>
<td>4.34</td>
<td>2.83</td>
</tr>
</tbody>
</table>

\[ X = \text{Ab} - 2 \text{En} - 5.5 \text{Fs} \]

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
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<td>35.53</td>
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<td>II †</td>
<td>III †</td>
<td>IV †</td>
</tr>
</tbody>
</table>

24
rifts and modern rifts is characterized by almost one and the same basaltic associations with quite similar mean compositions. A vast expansion of volcanism in precambrian time, as reported in recent publications [Engel and Engel, 1972; Condie, Baragar, 1974] shows that 3.5-2.5 billion years ago the oceans had perhaps as much distribution as in post-Permian time. That these two periods were both times of an active rift formation is understandable because rift and ocean genesis are interconjugate geological processes. From here it apparently follows that four basaltic groups have been existing always. But if we will check how the petrochemical groups of table 1 are distributed according to frequency of appearance, we will find that basalts of ancient rift-zones are not completely analogous to modern ones. Seventy percent of the studied basalt samples of ancient rift-zones were classified as olivin-hypersthene tholeiites. Moreover, traps of the Siberian platform and splits of the Ural Mts., which were taken for comparison also fall into the same group. Also 10% of ancient rift-zone basalts are related to the weak-alkalic group I. Such a distribution of frequencies is not characteristic of any modern rift zone. This complexity shows that the available data for precambrian rifts is unsufficient. As additional information is gathered we expect to reexamine this problem.

Let us consider now the volcanism of modern rift zones in different stages of their development. In this case we have an opportunity to trace the change in the character of volcanism during the process of rifting. It is possible to emphasize three stages in the development of volcanism in rift zones, coinciding with three stages of rift formation.
and which is totality comprise the lateral evolulional sequence.

The first stage is characterized by scattered volcanism without clear correlation with structure. The thick plateau basalts composed of undifferentiated alkali olivine basalts (group I) were formed during the fissure eruptions. Examples are the trap series of the Ethiopian rift or the Miocene basalts of the south-western part of the Baikal rift.

In the second stage occurs the formation of rift depression, with associated tensional thinning of the crust. Volcanism is located just at the rift's limit, predominantly in the lateral margins. Along with fissure eruptions of alkali olivine basalts, volcanoes of central type play the special role yielding differentiated basalts and trachytes (zone Vonje in Ethiopia, Chara rift in Baikal region, Siberia).

Two cases arise, depending on the extent of thinning of the crust:

1. The crust is relatively thick, 25-35 km; in this connection the petrochemistry of basalts does not change and we could distinguish volcanics of the first stage from those of the second according to the degree of completeness and strength of expression of the "Bowen trend" (for example, in the Baikal rift, group II).

2. The crust is relatively thin, 10-25 km; the localization of volcanism is expressed more clearly than in case 1, and the quartz tholeiites appear (quarternary volcanism of the Ethiopian rift, group III). The third stage is characterized by the complete rifting and separation of continental crust and volcanism now developing in oceanic crust, where olivine-hypersthene tholeiites are formed during the fissure eruptions (group IV). In addition, central-type volcanoes and their series of alkali
olivine basalts to trachytes develop on the islands of mid-oceanic ridges (group II).

**Suggested model.**

The author believes that the above mentioned three stages in the evolution of rift zones should be correlated with different positions of the zone of generation of basalt magma. The existence of the four basaltic groups should be sought in the peculiarities of processes running their course both in this zone and during the magma uplift to the earth's surface. According to data of many investigators such a zone is a "low-velocity layer", the formation of which apparently originated with partial melting of the upper mantel (Cook, 1961, 1967; Lambert, Willie, 1970).

Magneto-telluric survey data indicates that the "low-velocity layer" has a high thermal and electrical conductivity. Seismic data about the damping decay of transverse waves testifies in support of the hypothesis of near-melting conditions of the mantel there. Evaluation of heat flow allows one to determine the temperature in the roof of the "low-velocity layer" to be 1000-1200°C. It is very important that the physical properties of this layer are the same for continental and oceanic rift zones. The difference, consequently, points to the dissimilarity of depth of the top of the "low-velocity layer" which, for mid-oceanic rifts 5-7 km down and for continental rifts is 20-40 km.

The above four basaltic associations could be formed by the successive melting of lherzolite. (O'Hara, 1968, Kushiro and oth., 1968).

Depending on the depth to the "low-velocity layer", the magmatic source location, and whether we have dry or wet melting of lherzolite, alkali
olivine basalts, quartz tholeiites or olivine-hypersthene tholeiites could result.

We suggest the following model of this process, which takes into account evolution of the rift zone itself and also the evolution of volcanism within its limits.

For the first stage of development of the rift zone let us accept that in the "low-velocity layer" at depth 45-60 km, under lithostatic pressure 10-20 kb, there occurs partial melting (on the 15% level) of lherzolite, containing 0.1% H$_2$O. Both upper-wet-melting (pressure 10 kb, temperature near 1100-1150$^\circ$) and lower-dry-melting (pressure 20 kb, temperature near 1250$^\circ$) can produce derivative magmas of alkali olivide basalts. These magmas under quite tectonic conditions at the time of uplifting to the surface produce a differentiated series of alkaline olivine basalts (group II, olivine 3%, nepheline 5%, alumina content is quite high) or a non-differentiated series of olivine basalts (group I, olivine 14%, nepheline in the accessory amount, alumina content is average). The differences between these series result from the speed of magma uplifting and variations in volatile content.

The second stage of development of the rift zones is characterized by the approach of the "low-velocity layer" to the surface. The lower pressure limit, in which melting of lherzolite with 1% H$_2$O is geologically reasonable is 8 kb and the temperature must be 1300$^\circ$. Such temperature requires the unusually high thermal conductivity of order 10 ccal/grad. Under these conditions, low-alumina quartz tholeiites, containing 15% normative hypersthene could be melted from lherzolite (group III).
However, more ordinary thermodynamic conditions of quartz tholeiites generation are reached at 12 kb pressure and a temperature of 1350°.

At the third stage of the rift formation the "low-velocity layer" lies directly beneath the basaltic layer of the oceanic floor. With a high thermal gradient (500-1000 grad/km) and strong enrichment of volatiles, the olivine-hypersthene tholeiites could be generated from lhezzolite at 5-7 kb pressure and a temperature of 1300° (group IV).

Conclusions

Our data point to the following conclusions:

1. Cluster analysis was found to be an effective tool for multivariate classification (grouping) of rift basalt samples. Inductive classification types and groups were obtained on the basis of the petrochemical similarity of input samples. Apparently, geological objects which fall into the same group, besides their similarity, have an identical evolitional history.

2. Multivariable principal component analysis enabled us to detect the main petrochemical bi-polar tendency like $\frac{SiO_2 Al_2 O_3 K_2 O Na_2}{FeO MgO CaO TiO_2}$ which arises in the course of the pre-crystallization acid-basic differentiation of elements and fractional crystallization. We related this tendency to the "Bowen trend" and suggested using its very existence, its contribution in the total petrochemical dispersion (i.e. the weight of the first principal component) and its completeness as a qualitative diagnostic criterion for distinguishing basaltic series and as their important characteristic. It follows from our data that the strongest and most complete "Bowen trend" is characterized for alkali olivine basalts,
the weakest for tholeiitic basalts.

3. The existence of the established four groups of rift zones basalts is apparently manifested for the whole geological history of the earth, at least, since early Proterozoic. The controversial frequency of distribution of the ancient rift basalts do not allow us to conclude that past conditions of generation, differentiation and eruption of basaltic magma differ principally from those in our time.

4. An evolutionary sequence in the development of volcanism in modern rift zones is delineated. The gradual uplift of the "low-velocity layer" defines three successive stages in rift evolution. The character of basaltic rift volcanism may be explained not so much by development of the rift in the ocean vs. in a continental, as by the depth to the roof of the "low-velocity layer".

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References


