NONPARAMETRIC ESTIMATORS OF EFFECT SIZE
IN META-ANALYSIS

BY

LARRY V. HEDGES and INGRAM OLKIN

TECHNICAL REPORT NO. 193
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ABSTRACT

Quantitative research synthesis usually involves combining estimates of effect magnitude from several studies. The standardized mean difference is perhaps the most widely used index of effect magnitude in meta-analysis. When the observations are normally distributed the sample standardized mean difference is a consistent and asymptotically efficient estimator of effect size. However, in some cases the observations are far from normally distributed and a nonparametric index of effect magnitude is desirable. One such nonparametric estimator of effect size was introduced by Kraemer and Andrews. The present paper extends the logic of the Kraemer-Andrews estimator to provide related nonparametric estimators that estimate different parameters and which are appropriate under different experimental conditions.

Key Words: nonparametric estimators, meta-analysis, research synthesis, effect size.

Running Head: Nonparametric Estimators of Effect Size
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Quantitative methods are becoming increasingly important supplements to qualitative methods used in research reviewing (meta-analysis). One approach relies on the combination of quantitative indices of effect magnitude derived from different studies. This involves the extraction of an estimate of effect magnitude from each study. Glass (1976) suggested the standardized mean difference as the most suitable index of effect magnitude for studies examining the effect of an experimental treatment. These estimates are combined to obtain an estimate of the overall average effect size. In addition, the investigator may wish to determine if variations among effect sizes are related to variations in experimental conditions across studies.

Quantitative syntheses of research using effect sizes have been used with many collections of research studies since the first behavioral science meta-analysis of studies on the effects of psychotherapy (Smith and Glass, 1977). Recent meta-analyses have included studies of interpersonal expectancy effects (Rosenthal and Rubin, 1978), the effects of class size on achievement and classroom processes (Glass and Smith, 1979; Smith and Glass, 1980), sex differences in conformity (Eagly and Carli, 1980), sex differences in spacial ability (Hyde, 1981), the effects of teaching methods in higher education (e.g., Kulik, Kulik, and Cohen, 1979), and the effects of informal or open teaching (Giaconia and Hedges, 1982). It should be noted that research synthesis has an older history in the context of agricultural experiments (e.g., Cochran, 1937).
Interest in quantitative synthesis of research using effect sizes has led to a growing body of work on statistical properties of effect sizes and to the development of statistical methods for fitting explanatory models to variations in effect sizes from a series of studies. Formulas for converting various test statistics to effect size estimates were given by Glass, McGaw, and Smith (1981). The development of formal statistical theory for effect size estimates depends on the underlying statistical model used, such as the assumptions for Student's $t$ or Snedecor's $F$ tests. Thus we might assume that the observations in a study are independently and normally distributed about experimental and control group means $\mu^E$ and $\mu^C$ with common variance $\sigma^2$. The population effect size $\delta$ is the standardized mean difference

$$\delta = (\mu^E - \mu^C)/\sigma.$$  

Glass (1976) suggested estimating the population effect size $\delta$ by using the sample standardized mean difference (sample effect size)

$$g = (\bar{x}^E - \bar{x}^C)/s,$$

where $\bar{x}^E$ and $\bar{x}^C$ are the experimental and control group sample means and $s^2$ is a sample estimate of the within group variance. The sampling distribution of $g$ and a large sample normal approximation was obtained by Hedges (1981), who also showed that $g$ was biased and obtained an unbiased estimator of $\delta$ that has smaller variance than $g$. Procedures for combining estimates of effect size from $k$ independent studies were studied
by Hedges (1982a) who showed that a simple weighted average was optimal (asymptotically efficient). He obtained the large sample distribution of the weighted average and showed (via simulation studies) that the large sample distribution is quite accurate for the effect sizes and sample sizes usually encountered in meta-analysis. Hedges also obtained a test that \( k \) studies share a common underlying population effect size. A similar test of homogeneity of effect size was obtained independently by Rosenthal and Rubin (1982). Statistical procedures have also been developed that enable the investigator to determine the relationship between characteristics of studies and their effect sizes. One procedure permits the investigator to fit models with discrete independent variables to effect sizes and constitutes an analogue to the analysis of variance for effect sizes (Hedges, 1982b). Other procedures permit the investigator to fit general linear models to effect sizes (Hedges, 1982c) or correlation coefficients (Hedges and Olkin, 1983). A natural test of goodness of fit of the explanatory model is available for each of these procedures.

The usual model (that assumptions of the \( t \) test are met in each study) is often convenient, but not always realistic. Several examples of real data that do not meet the assumptions of the usual model are cited by Kraemer and Andrews (1982). They suggest that in many cases, the data is skewed or otherwise requires a transformation to achieve even approximate normality. In such cases the effect sizes calculated from the data before and after transformation may differ substantially. One approach is to use the effect size calculated on the transformed data which, presumably, is reasonably normal. An alternative is to develop an estimator of the effect size that
is unaffected by monotonic transformations of the observations. This was the motivation for the nonparametric estimator of effect size proposed by Kraemer and Andrews (1982).

The usual model for the observations in each study (assumptions of the \( t \) test) may also be invalid if some of the observations are outliers. The presence of outliers distorts the values of test statistics and likely has a similar effect on estimates of effect size. One approach is to eliminate outliers from the data before the data analysis and calculate the effect size estimate on the (outlier free) data used in the statistical analysis. An alternative approach is to develop methods for estimating effect size that are insensitive to the presence of outliers.

The present paper provides an examination of some alternatives to parametric estimators of effect size. Although the estimators are nonparametric in the sense that they do not explicitly depend on the parametric structural model for the observations, they may not be distribution-free. We begin by discussing estimators of effect size derived by using robust estimators of \( \bar{\mu}^E \), \( \mu^C \), and \( \sigma \), and then examine the nonparametric estimator of effect size proposed by Kraemer and Andrews, (1982). This suggests a number of alternative estimators which are illustrated on the data provided by Kraemer and Andrews.

**Estimates of Effect Size that are Robust Against Outliers**

The effect size \( \delta \) is the ratio of a mean difference to a standard deviation. To obtain an estimator of effect size robust against outliers, we estimate the mean difference and the standard deviation by robust
estimators. Many such estimators are possible. For example, the mean of each group can be estimated by the median, and the standard deviation of each group can be estimated by the range or other linear combination of order statistics, that is

\[
\hat{\delta} = \frac{\text{Med}(x^E) - \text{Med}(x^C)}{\tilde{\sigma}},
\]

where \(\text{Med}(x^E)\) and \(\text{Med}(x^C)\) are the medians of the observations in the experimental and control groups respectively, and

\[
\tilde{\sigma} = a_2x^{C(2)} + \cdots + a_{n-1}x^{C(n-1)}.
\]

where \(x^{C(1)} \leq x^{C(2)} \leq \cdots \leq x^{C(n-1)} \leq x^{C(n)}\) are the ordered values in the control group. The coefficients \(a_2, \ldots, a_{n-1}\) are to be chosen optimally; they are not easily described but are tabulated (see e.g., Sarhan and Greenberg (1962) p. 218-251). Note that the smallest and largest values have been omitted in the estimator of \(\sigma\).

Since the estimate \(\hat{\delta}\) of \(\delta\) given in (3) does not involve the most extreme observations, it is robust against shifts of the most extreme observations.

**Nonparametric Estimator of Effect Size**

A nonparametric approach to estimating effect size was suggested by Kraemer and Andrews (denoted KA). Their technique requires that both pretest \((x)\) and posttest \((y)\) scores be available for each individual in the experimental and control groups of an experiment. Calculation of the effect size can be illustrated by reference to a table of pretest and posttest scores
for the experimental and control groups (see Table 1). We first review the rationale for the KA estimator; this serves to clarify how several different estimators can be constructed.

[TABLE 1 HERE]

For each group determine the sample proportion, $p$, of $x$ scores that lie below the median $y$ score. This proportion (for the control and treatment group) corresponds to a standard normal deviate, $\hat{\delta}$, that is,

$$\Phi(\hat{\delta}) = p \quad \text{or} \quad \hat{\delta} = \Phi^{-1}(p),$$

where $\Phi(x)$ is that standard normal cumulative distribution function. An application of this procedure to both the experimental and control group scores yields posttest versus pretest effect for $t$ size estimates $\hat{\delta}^E$ and $\hat{\delta}^C$, respectively, from which an overall effect size $\hat{\delta}$ is obtained as

$$\hat{\delta} = \hat{\delta}^E - \hat{\delta}^C.$$

The key idea in the above development is to estimate effect size by using the proportion of $x$ scores that lie below the "middle" of the $y$ score distribution. However, different definitions of the middle of the $y$ score distribution can be used, and these would result in slightly different estimators. For simplicity, we describe the procedure using the median as the definition of the center of the posttest score distribution.

Remark. Kraemer and Andrews use a slightly different definition of the center because they are interested in the pre to posttest change of individuals near the center of the pretest score distribution. They define a "critical prerange" as the $x$ median plus the nearest two $x$ scores on either side of the median and
then define the center of \( y \) scores as the median of the 5 posttest scores corresponding to the critical prerange. Once the middle of the \( y \) score distribution is defined, their procedure proceeds as described previously.

An example will clarify the procedure. Some data on systolic blood pressure given in Kraemer and Andrews (1982) is reproduced in Table 1. The pretreatment (\( x \)) and posttreatment (\( y \)) scores of experimental and control group subjects are arranged in ascending order of pretest scores. The median of the \( y \) scores in the control group is 154.5. The number of \( x \) scores less than 154.5 is 11, so that

\[
\delta^C = \Phi^{-1}(11/20) = 0.126
\]

In a similar manner we find that \( \hat{\delta}^E = \Phi^{-1}(0) \). In such a case, Kraemer and Andrews recommend using \( 1/(n+1) \) instead of zero, so that \( \hat{\delta}^E = \Phi^{-1}(0.048) = -1.665 \). Combining \( \hat{\delta}^E \) and \( \delta^C \) we obtain an estimate \( \hat{\delta} \) of the effect size:

\[
\hat{\delta} = \hat{\delta}^E - \delta^C = 1.665 - .126 = -1.791
\]

Remark. The result obtained by Kraemer and Andrews using slightly different definitions of \( \delta^E \) and \( \delta^C \) is -1.85.

Estimators Based on Differences of Control Versus Treatment Proportions

The logic of nonparametric estimators of effect size is best illustrated by examining bivariate plots of the pretest (\( x \)) scores versus posttest (\( y \)) scores (see Fig. 1). The KA procedure as applied to each group consists in looking at the proportion \( p \) of points (\( x,y \)) that such that \( x \) is less than the median of \( y \), i.e. calculating the proportion of points to the
left of line $L_1$. We then use the inverse normal cumulative distribution function $\phi^{-1}$ to obtain an estimate of pre-post effect size from $p$.

The KA estimator is

$$\hat{\delta}_1 = \phi^{-1}(p_1^E) - \phi^{-1}(p_1^C).$$

When the observations are normally distributed, $\hat{\delta}_1$ estimates

$$\frac{E - E}{\sigma_x} = \frac{C - C}{\sigma_x},$$

where $\mu_y, \mu_x, \sigma_x$ are the experimental group posttest mean, pretest mean, and pretest standard deviation respectively, and $\mu_C, \mu_x, \sigma_x$ are the analogous parameters for the control group.

[FIGURE 1 HERE]

Other regions of the bivariate plot can be used to derive estimates of effect size. Indeed, the KA procedure is only one of several natural estimators of effect size obtained by using proportions of points falling into different regions of the bivariate plot. The rationale for the KA procedure is that the larger the treatment effect, the smaller the proportion of pretest scores that exceed the posttest median.

A related estimator $\hat{\delta}_2$ is based on the proportion $p_2^C$ of the posttest scores in the control group that exceed the pretest median in the control group, and the proportion $p_2^E$ of the posttest scores in the experimental group that exceed the pretest median in the experimental group. For each group the relevant proportions correspond to the proportion of observations below the line $L_2$. This estimator is defined by

$$\hat{\delta}_2 = \phi^{-1}(p_2^E) - \phi^{-1}(p_2^C).$$

For the Kraemer-Andrews data $p_2^E = .75$ and $p_2^C = .50$, Therefore $\hat{\delta}_2 = \phi^{-1}(.75) = .675$, $\hat{\delta}_2^C = \phi^{-1}(.50) = 0.0$, and $\hat{\delta}_3 = \hat{\delta}_2^E - \hat{\delta}_2^C = .675$. 
When the observations are normally distributed, \( \hat{\delta}_2 \) estimates
\[
\frac{E - \mu_x}{\sigma_x} - \frac{C - \mu_x}{\sigma_x},
\]
where \( \sigma_x \) and \( \sigma_y \) are experimental and control group posttest standard deviations.

Still another alternative can be considered. Let \( p_3^C \) denote the proportion of individuals in the control group whose scores increase from pretest to posttest. This corresponds to the proportion of observations below the line \( L_3 \). If \( p_3^E \) is the proportion of individuals whose scores increase from pretest to posttest in the experimental group, the estimator \( \hat{\delta}_3 \) is given by
\[
\hat{\delta}_3 = \Phi^{-1}(p_3^E) - \Phi^{-1}(p_3^C).
\]

In the Kraemer-Andrews data \( p_3^E = .85 \) and \( p_3^C = .50 \). Therefore \( \hat{\delta}_3^E = \Phi^{-1}(.85) = 1.038 \), \( \hat{\delta}_3^C = \Phi^{-1}(.50) = 0.0 \), and \( \hat{\delta}_3 = \hat{\delta}_3^E - \hat{\delta}_3^C = 1.038 \).

When the scores are normally distributed, \( \hat{\delta}_3 \) estimates
\[
\frac{E - \mu_x}{\sigma_x} - \frac{C - \mu_x}{\sigma_x},
\]
where \( \mu_x \), \( \mu_y \), and \( \mu_x \) are experimental and control group population means on the posttest and pretest respectively, and
\[
(\sigma_x^2) = (\sigma_y^2) + (\sigma_x^2) - 2\sigma_x \rho_{xy},
\]
\[
(\sigma_x^2) = (\sigma_y^2) + (\rho_{xy})^2 - 2\sigma_x \rho_{xy},
\]
and \( \rho_{xy} \) and \( \rho_{xy} \) are the correlations between pretest scores and posttest scores in the control and experimental groups respectively. Note that \( \hat{\delta}_3 \)
estimates the difference between standardized mean gains in the experimental and control groups.

The main distinction between the estimators $\hat{\delta}_1$, $\hat{\delta}_2$, $\hat{\delta}_3$ lies in the normalizations. Each estimator estimates a parameter of the form

$$\frac{\mu_y - \mu_x}{\sigma_y} - \frac{\mu_y - \mu_x}{\sigma_x},$$

where the standard deviations are $\sigma_x$, $\sigma_y$ and $\sigma_x$, for $\hat{\delta}_1$, $\hat{\delta}_2$, $\hat{\delta}_3$ respectively. Thus, the particular $\hat{\delta}$ that should be used depends on the normalization desired.

**Estimators Based on Gain in the Experimental Relative to the Control Group**

A class of estimators can be derived by using the proportion of scores in the control group that gain more than would be expected in the experimental group. This idea is illustrated graphically in the figure. Determine the medians of the pretest and posttest scores in the experimental group, and on the control group data draw a line $M$ from the origin $(0,0)$ through the point $(\text{Med}(x^E), \text{Med}(y^E))$. The equation of the line $M$ is

$$y^C = \frac{\text{Med}(y^E)}{\text{Med}(x^E)} x^C.$$

The proportion, $q_1$, of points below line $M$ represents the proportion of people in the control group whose scores increased relatively more than their experimental group counterparts. Then the estimator $\hat{\gamma}_1$ of effect size is given by
\( \hat{\gamma}_1 = \Phi^{-1}(q_1) \).

For the data in Table 1, \( q_1 = .80 \) and \( \hat{\gamma}_1 = \Phi^{-1}(.80) = .842 \).

If the scores are normally distributed, \( \hat{\gamma}_1 \) estimates \( (\mu_y^E - \mu_y^C)/\sigma_y^C \).

A symmetric version is obtained by drawing on the plot of experimental data a line between the origin \((0,0)\) and the point \((\text{Med}(x^C), \text{Med}(y^C))\) representing the medians of the posttest scores in the control group. Define \( q_2 \) as the proportion of scores in the experimental group that lie below the line, that is, for which

\[
y^E < \text{Med}(y^C) \frac{x^E}{\text{Med}(x^C)}.
\]

The estimator \( \hat{\gamma}_2 \) of effect size is

\( \hat{\gamma}_2 = \Phi^{-1}(q_2) \).

For the data in Table 1, \( q_2 = .90 \) and \( \hat{\gamma}_2 = \Phi^{-1}(.90) = 1.282 \).

If the scores are normally distributed then \( \hat{\gamma}_2 \) estimates \( (\mu_y^E - \mu_y^C)/\sigma_y^E \).

Nonparametric Estimates Involving Only Posttest Scores

If random assignment is made to control and treatment groups then one expects the distribution of pretest scores to be similar in both groups. This suggests using only the posttest scores in the experimental and control groups. (This procedure does have the advantage that it can be used when no pretest scores are available.) Calculate the proportion \( q_1^* \) of control groups scores less than the experimental group median (below line \( M^* \)) and transform the proportion to an estimate, \( \gamma_1^* \), of effect size.
\[ \hat{\gamma}_1^* = \phi^{-1}(q_1^*) \]

Examining Figure 1 we see that \( q_1^* = .90 \) for the Kraemer-Andrews data, which corresponds to \( \hat{\gamma}_1^* = \phi^{-1}(.90) = 1.282 \).

When the observations are independently normally distributed \( \hat{\gamma} \) is an estimator of \( (\mu_E^y - \mu_C^y)/\sigma_y^C \), the mean difference standardized by the standard deviation \( \sigma_y^C \) of the control group scores.

A symmetric estimator is obtained by using the proportion \( q_2^* \) of scores in experimental group that exceed the median score in the control group. The proportion \( q_2^* \) is transformed into an estimator of effect size via

\[ \hat{\gamma}_2^* = \phi^{-1}(q_2^*) \]

The Kraemer-Andrews data given in Table 1 has \( q_2^* = .90 \), which corresponds to \( \hat{\gamma}_2^* = \phi^{-1}(.90) = 1.282 \).

When the observations are independently normally distributed, \( \hat{\gamma}_2^* \) is an estimator of \( (\mu_E^y - \mu_C^y)/\sigma_y^E \), the experimental-control group mean difference standardized by the standard deviation \( \sigma_y^E \) of the scores in the experimental group. Therefore if the scores are normally distributed and the experimental and control group standard deviations are equal, both \( \hat{\gamma}_1^* \) and \( \hat{\gamma}_2^* \) estimate the same quantity. We would use \( \hat{\gamma}_1 \) rather than \( \hat{\gamma}_1^* \) when both pre and posttest are available, and use \( \hat{\gamma}_1^* \) only when post-test data is available.

**Relationships Between Estimators**

The seven nonparametric estimators of effect size are different in the sense that they use the data differently and they estimate slightly different parameters. The proportions used to calculate each of the estimators is given in Table 2 along the quantity estimated under the assumption that the observations are normally distributed. Note that \( \hat{\gamma}_1 \) and \( \hat{\gamma}_1^* \) estimate the
same quantity, and that $\hat{\gamma}_2$ and $\hat{\gamma}^\prime_2$ estimate the same quantity. All
four estimators ($\hat{\gamma}_1$, $\hat{\gamma}^\prime_1$, $\hat{\gamma}_2$, and $\hat{\gamma}^\prime_2$) estimate identical quantities if
$\sigma^E_y = \sigma_C^y$. Similarly $\hat{\delta}_1$ and $\hat{\delta}_2$ estimate the same parameter if
$\sigma^E_x = \sigma_C^x$ and $\sigma^E_y = \sigma_C^y$.

[TABLE 2 HERE]
TABLE 1. Systolic blood pressure data on pre and post treatment and control data from 20 hypertensives.

<table>
<thead>
<tr>
<th>Treatment Group</th>
<th>Control Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>pre</td>
<td>post</td>
</tr>
<tr>
<td>134</td>
<td>130</td>
</tr>
<tr>
<td>135</td>
<td>131</td>
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<tr>
<td>135</td>
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<td>179</td>
<td>149</td>
</tr>
<tr>
<td>180</td>
<td>150</td>
</tr>
</tbody>
</table>

Medians 153 133 154 154.5
<table>
<thead>
<tr>
<th>Estimator</th>
<th>Proportion</th>
<th>Quantity Estimated under Normality Assumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\delta}_1^C$</td>
<td>$x^C &lt; \text{Med}(y^C)$</td>
<td>$(\mu_y^C - \mu_x^C)/\sigma_x^C$</td>
</tr>
<tr>
<td>$\hat{\delta}_1^E$</td>
<td>$x^E &lt; \text{Med}(y^E)$</td>
<td>$(\mu_y^E - \mu_x^E)/\sigma_x^E$</td>
</tr>
<tr>
<td>$\hat{\delta}_1 = \hat{\delta}_1^E - \hat{\delta}_1^C$</td>
<td></td>
<td>$\delta_1^E - \delta_1^C$</td>
</tr>
<tr>
<td>$\hat{\delta}_2^C$</td>
<td>$y^C &lt; \text{Med}(x^C)$</td>
<td>$(\mu_y^C - \mu_x^C)/\sigma_x^C$</td>
</tr>
<tr>
<td>$\hat{\delta}_2^E$</td>
<td>$y^E &lt; \text{Med}(x^E)$</td>
<td>$(\mu_y^E - \mu_x^E)/\sigma_x^E$</td>
</tr>
<tr>
<td>$\hat{\delta}_2 = \hat{\delta}_2^E - \hat{\delta}_2^C$</td>
<td></td>
<td>$\delta_2^E - \delta_2^C$</td>
</tr>
<tr>
<td>$\hat{\delta}_3^C$</td>
<td>${y^C &lt; x^C}$</td>
<td>$(\mu_y^C - \mu_x^C)/\sigma_x^C$</td>
</tr>
<tr>
<td>$\hat{\delta}_3^E$</td>
<td>${y^E &lt; x^E}$</td>
<td>$(\mu_y^E - \mu_x^E)/\sigma_x^E$</td>
</tr>
<tr>
<td>$\hat{\delta}_3 = \hat{\delta}_3^E - \hat{\delta}_3^C$</td>
<td></td>
<td>$\delta_3^E - \delta_3^C$</td>
</tr>
<tr>
<td>$\hat{\gamma}_1$</td>
<td>${y^C &lt; \frac{\text{Med}(y^E)}{\text{Med}(x^E)} x^C}$</td>
<td>$(\mu_y^E - \mu_y^C)/\sigma_y^C$</td>
</tr>
<tr>
<td>$\hat{\gamma}_2$</td>
<td>${y^E &lt; \frac{\text{Med}(y^C)}{\text{Med}(x^C)} x^E}$</td>
<td>$(\mu_y^E - \mu_y^C)/\sigma_y^E$</td>
</tr>
<tr>
<td>$\hat{\gamma}_1^*$</td>
<td>$y^C &lt; \text{Med}(y^E)$</td>
<td>$(\mu_y^E - \mu_y^C)/\sigma_y^C$</td>
</tr>
<tr>
<td>$\hat{\gamma}_2^*$</td>
<td>$y^E &lt; \text{Med}(y^C)$</td>
<td>$(\mu_y^E - \mu_y^C)/\sigma_y^E$</td>
</tr>
</tbody>
</table>
Figure 1

\[ \hat{\gamma}_1 = \phi^{-1} \left( \frac{16}{20} \right) = 0.842 \]

\[ \hat{\delta}_1 = \phi^{-1} \left( \frac{11}{20} \right) = 0.126 \]

\[ \hat{\gamma}_2 = \phi^{-1} \left( \frac{18}{20} \right) = 1.282 \]

\[ \hat{\delta}_2 = \phi^{-1} \left( \frac{10}{20} \right) = 0.00 \]

\[ \hat{\delta}_3 = \phi^{-1} \left( \frac{10}{20} \right) = 0.00 \]
References


