A TEST FOR THE MEAN OF A MULTIVARIATE NORMAL DISTRIBUTION
WITH UNKNOWN COVARIANCE MATRIX
AGAINST ONE-SIDED ALTERNATIVES

BY
MICHAEL D. PERLMAN

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1. Introduction.

Let $X_1, \ldots, X_N$ be independent, identically distributed random $p$-dimensional vectors, each having a multivariate normal distribution with unknown mean vector $\mu$ and unknown covariance matrix $\Sigma$. It is known a priori that $\mu$ lies in the positive orthant of $p$-dimensional Euclidean space $\mathbb{R}^p$, i.e., $\mu_i \geq 0$, $i = 1, \ldots, p$, where $\mu' = (\mu_1, \ldots, \mu_p)$. We abbreviate this by writing $\mu \geq 0$. We wish to test the hypothesis $H: \mu = 0$ against the "one-sided" alternative $K = \mu \geq 0$, $\mu \neq 0$.

Nüesch (1966) attempts to derive the likelihood ratio test (LRT) for this problem, but unfortunately his work is in error. It is the purpose of this note to derive the LRT (Theorem 3.1) and obtain its distribution under $H$ (Theorem 4.2).

For the case $\Sigma$ known (Section 2), power calculations by Bartholomew (1961) and Nüesch (1964) show that the LRT is substantially more powerful than the usual chi-square test based on $\bar{X}'\Sigma^{-1}\bar{X}$.

Throughout this paper the letters $c$ and $d$ represent arbitrary positive constants.

2. Preliminaries. The Case $\Sigma$ Known.

The problem of testing $H$ against $K$ when $\Sigma$ is known has been treated by Bartholomew (1961), Kudô (1963), and Nüesch (1966). We outline the derivation of the LRT in this case because some of the results are needed to treat the case where $\Sigma$ is unknown.
The likelihood function is

\[(2.1) \quad L = c \cdot |\Sigma|^{-N/2} \exp\left[-\frac{1}{2} \text{tr} \Sigma^{-1} S - \frac{1}{2} N(\overline{X} - \mu)'\Sigma^{-1}(\overline{X} - \mu)\right],\]

where \(S = \sum_{\alpha=1}^{N} (X_\alpha - \overline{X})(X_\alpha - \overline{X})'\). When \(\mu = 0\),

\[(2.2) \quad \max_{\{\mu=0\}} L = c \cdot |\Sigma|^{-N/2} \exp\left[-\frac{1}{2} \text{tr} \Sigma^{-1} S + \frac{1}{2} N(\overline{X})'\Sigma^{-1}\overline{X}\right].\]

Let \(\hat{\mu}_\Sigma\) denote the maximum likelihood estimator of \(\mu\) under the restriction \(\mu \geq 0\). We write \(\hat{\mu}_\Sigma\) to emphasize its dependence on \(\Sigma\).

If \(\overline{X} \geq 0\), then \(\hat{\mu}_\Sigma = \overline{X}\). If \(\overline{X}\) does not lie in the positive orthant, then \(\hat{\mu}_\Sigma\) is the unique vector in the positive orthant satisfying

\[(2.3) \quad \min_{\{\mu \geq 0\}} (\overline{X} - \mu)'\Sigma^{-1}(\overline{X} - \mu) = (\overline{X} - \hat{\mu}_\Sigma)'\Sigma^{-1}(\overline{X} - \hat{\mu}_\Sigma).\]

Therefore,

\[(2.4) \quad \max_{\{\mu \geq 0\}} L = c \cdot |\Sigma|^{-N/2} \exp\left[-\frac{1}{2} \text{tr} \Sigma^{-1} S + \frac{1}{2} N(\overline{X} - \hat{\mu}_\Sigma)'\Sigma^{-1}(\overline{X} - \hat{\mu}_\Sigma)\right].\]

From (2.2) and (2.4) we obtain the following form of the LRT: Reject \(H\) if

\[(2.5) \quad N\overline{X}'\Sigma^{-1}\overline{X} - N(\overline{X} - \hat{\mu}_\Sigma)'\Sigma^{-1}(\overline{X} - \hat{\mu}_\Sigma) \geq c .\]

The following lemma appears in Kudô (1963) and Nüesch (1966):

**Lemma 2.1.** \(\hat{\mu}_\Sigma\) is the unique vector such that

\[(2.6) \quad \Sigma^{-1}(\hat{\mu}_\Sigma - \overline{X}) \geq 0 ,\]

\[(2.7) \quad (\sigma_{j1}, \ldots, \sigma_{jp})(\hat{\mu}_\Sigma - \overline{X}) > 0 \quad \text{implies} \quad \hat{\mu}_{\Sigma, j} = 0 ,\]

where \(\Sigma^{-1} = (\sigma_{jk})\) and \(\hat{\mu}_{\Sigma, j}\) is the \(j\)-th component of \(\hat{\mu}_\Sigma\).

This lemma immediately yields
\[(\mu_{\Sigma} - \bar{X})'\Sigma^{-1}\mu_{\Sigma} = 0,\]

which simply states that \(\mu_{\Sigma} - \bar{X}\) is orthogonal to \(\mu_{\Sigma}\), where orthogonality is determined by the inner product \((x,y) = x'\Sigma^{-1}y\). Using (2.8) the LRT (2.5) becomes: Reject \(H\) if

\[(2.9) \quad N_{\mu_{\Sigma}'\Sigma^{-1}\mu_{\Sigma}} \geq c.\]

Kudo (1963) and Müesch (1966) have obtained the distribution of the LRT statistic (2.9) in the following form when \(H\) is true:

\[(2.10) \quad P[N_{\mu_{\Sigma}'\Sigma^{-1}\mu_{\Sigma}} \geq c] = \sum_{k=1}^{p} P[\chi^2_k \geq c] w(p,k,\Sigma),\]

where \(\chi^2_k\) denotes a chi-square variate with \(k\) degrees of freedom and \(w(p,k,\Sigma)\) is the probability that exactly \(k\) of the \(p\) components of \(\mu_{\Sigma}\) are (strictly) positive. The weights \(w(p,k,\Sigma)\) are discussed in Section 4.

We conclude this section with the following lemma which is needed in Section 4. It is similar to lemmas appearing in Bartholomew (1961) and Kudo (1963).

**Lemma 2.2.** Let \(t\) be a random \(p \times 1\) vector in \(\mathbb{R}^p\) which possesses a density with respect to Lebesgue measure. Suppose that the distribution of \(t\) is orthogonally invariant, i.e., for any orthogonal transformation \(\Gamma\), \(t\) and \(\Gamma t\) are identically distributed. Then:

(i) \(t't\) is independent of \(t/(t't)^{1/2}\), the unit vector lying along \(t\).

(ii) \(t/(t't)^{1/2}\) is distributed uniformly over the unit sphere in \(\mathbb{R}^p\).
(iii) For any non-singular linear transformation $A$,
\[
P[At > 0] = \frac{m(A^{-1}(\mathcal{O}) \cap \mathcal{S})}{m(\mathcal{S})} = \psi(A^{-1}),
\]
where $\mathcal{S}$ is the unit sphere in $\mathbb{R}^p$, $\mathcal{O}$ is the positive orthant, $A^{-1}(\mathcal{O})$ is the image of $\mathcal{O}$ under $A^{-1}$, and $m$ is Lebesgue measure over $\mathcal{S}$.

Proof. (i) and (ii) are easily proved by transforming $t$ to polar coordinates, as in Section 3.1 of Bartholomew (1961). (iii) follows from (ii) by noting that $P[At > 0] = P[At/(t't)^{1/2} > 0] = P[t/(t't)^{1/2} \in (A^{-1}(\mathcal{O}) \cap \mathcal{S})]$. ||

Remark 1. As a consequence of (i), the events $\{t't \geq d\}$ and $\{At \geq 0\}$ are independent.

Remark 2. The condition of the lemma is satisfied in each of the following two cases:

\begin{align*}
(2.11) & \quad t \sim N(0, I), \text{ the normal distribution with} \\
& \quad \text{mean } 0 \text{ and covariance matrix } I \text{ (the identity matrix)}, \\
(2.12) & \quad t = S^{-1}y, \text{ where } S \text{ and } Y \text{ are independent,} \\
& \quad S \sim W(n, I) \text{ (the Wishart distribution with expectation } nI \text{ and } n \text{ degrees of freedom), and } Y \sim N(0, I).
\end{align*}

We show that the condition of the lemma is satisfied in (2.12) by writing $\Gamma t = \Gamma S^{-1}y = (\Gamma S\Gamma')^{-1}\Gamma Y$ and noting that $S$ and $Y$ are orthogonally invariant.

3. The Likelihood Ratio Test When $\Sigma$ is Unknown.

To obtain the LRT of $H$ versus $K$ when $\Sigma$ is unknown, we must find the maximum value of $L$ in (2.1) over each of the sets
\{\mu = 0, \Sigma > 0\} and \{\mu \geq 0, \Sigma > 0\}, where \Sigma > 0 denotes that \Sigma is a positive definite symmetric matrix. First, for fixed \mu, it is well-known (e.g., Anderson (1958), Lemma 3.2.2) that the maximum of \textbf{L} over the set \{\Sigma > 0\} is given by

\[(3.1) \quad \max_{\{\Sigma > 0\}} \textbf{L} = c \cdot |S + N(\overline{X}_\mu)(\overline{X}_\mu)'|^{-N/2}

= c \cdot |S|^{-N/2}[1 + N(\overline{X}_\mu)' S^{-1}(\overline{X}_\mu)]^{-N/2}.

The second equality follows from the fact that \text{ |I}_p + UU' = |I_q + U'U| for any \(p \times q\) matrix \(U\). Setting \mu = 0 in (3.1) we obtain

\[(3.2) \quad \max_{\{\mu = 0, \Sigma > 0\}} \textbf{L} = c \cdot |S|^{-N/2}[1 + N\overline{X}_\mu S^{-1}\overline{X}_\mu]^{-N/2}.

Next we must maximize (3.1) over the set \{\mu \geq 0\}. This is equivalent to minimizing \((\overline{X}_\mu)' S^{-1}(\overline{X}_\mu)\) over \{\mu \geq 0\}. Equation (2.3) implies that this minimum is attained when \mu = \hat{\mu}_S, so that

\[(3.3) \quad \max_{\{\mu \geq 0, \Sigma > 0\}} \textbf{L} = c \cdot |S|^{-N/2}[1 + N(\overline{X}_{\hat{\mu}_S})' S^{-1}(\overline{X}_{\hat{\mu}_S})]^{-N/2}.

Combining (3.1) and (3.3) we obtain the following form of the LRT:

Reject \ \textbf{H} if

\[
\frac{1 + N\overline{X}_{\hat{\mu}_S} S^{-1}\overline{X}_{\hat{\mu}_S}}{1 + N(\overline{X}_{\hat{\mu}_S})' S^{-1}(\overline{X}_{\hat{\mu}_S})} \geq c.
\]

Using (2.8) with \Sigma replaced by \textbf{S}, we see that

\[
\overline{X}' S^{-1}\overline{X} = (\overline{X}_{\hat{\mu}_S})' S^{-1}(\overline{X}_{\hat{\mu}_S}) + \hat{\mu}_S S^{-1}\hat{\mu}_S.
\]

Therefore, we have proved

\textbf{Theorem 3.1.} The LRT for the problem of testing \textbf{H}: \mu = 0, \Sigma > 0.
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$K_k: \mu(1) = 0$, $\mu(2) \neq 0$, where $\mu = (\mu(1), \mu(2))$ and $\mu(2)$ is $k \times 1$. The distribution of $U_k$ is derived in a straightforward manner (e.g., Stein (1966), part IV, pp. 7-11), and we can readily adapt this derivation to obtain the conditional distribution of $U$.

We now begin the evaluation of $P[U \geq c]$. Let $M$ be a subset of $\{1, \ldots, p\}$ and let

$$\mathcal{K}_{M,S} = \{ \bar{x}: \hat{\mu}_S, 1 > 0 \text{ for } i \in M, \hat{\mu}_S, i = 0 \text{ for } i \notin M \}.$$

Then

$$P[U \geq c] = \sum_M P[U \geq c, \bar{x} \in \mathcal{K}_{M,S}],$$

where the summation extends over all subsets $M$ of $\{1, \ldots, p\}$. To evaluate $P[U \geq c, \bar{x} \in \mathcal{K}_{M,S}]$ for fixed $M$ (and hence for fixed $k$), we may suppose without loss of generality that $M$ consists of the last $k$ members of $\{1, \ldots, p\}$. Write

$$\bar{x} = \left(\begin{array}{c} y \\ z \end{array}\right), \quad \hat{\mu}_S = \left(\begin{array}{c} \hat{\mu}(1) \\ \hat{\mu}(2) \end{array}\right), \quad S = \left(\begin{array}{cc} S_{11} & S_{12} \\ S_{21} & S_{22} \end{array}\right), \quad S^{-1} = \left(\begin{array}{cc} S_{11} & S_{12} \\ S_{21} & S_{22} \end{array}\right), \quad \Sigma = \left(\begin{array}{cc} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{array}\right),$$

where $z$ and $\hat{\mu}_{(2)}$ are $k \times 1$, $S_{22}$, $S_{22}$, and $\Sigma_{22}$ are $k \times k$. To characterize the event $\{\bar{x} \in \mathcal{K}_{M,S}\}$ we need the following lemma which appears in Muesch (1966) and Kudo (1965), although in the latter paper it assumes a somewhat different form.

Lemma 4.1. $\mathcal{K}_{M,S} = \{\bar{x}: S_{11}^{-1}y \leq 0, z - S_{21}S_{11}^{-1}y > 0\}$.

Proof. (i) (Muesch): First suppose that $\bar{x} \in \mathcal{K}_{M,S}$. Then by definition of $\mathcal{K}_{M,S}$, $\hat{\mu}_S = \left(\begin{array}{c} 0 \\ \hat{\mu}(2) \end{array}\right)$, where $\hat{\mu}_{(2)} > 0$. Hence by (2.7) of Lemma 2.1 we have
\[
(s^{21} \ s^{22}) \begin{pmatrix}
-\gamma \\
\uhat_{(2)} - Z
\end{pmatrix} = 0 ,
\]

or equivalently,
\[
(4.1) \quad Z + (s^{22})^{-1} s^{21} Y = \uhat_{(2)} > 0 .
\]

But \((s^{22})^{-1} s^{21} = -s^{21} s_{11}^{-1}\), so (4.1) becomes
\[
Z - s_{21} s_{11}^{-1} Y = \uhat_{(2)} > 0 .
\]

Next, (2.6) implies
\[
(s^{11} \ s^{12}) \begin{pmatrix}
-\gamma \\
\uhat_{(2)} - Z
\end{pmatrix} \geq 0 ,
\]

which, together with (4.1), yields
\[
[s^{11} - s^{12} (s^{22})^{-1} s^{21}] Y = s_{11}^{-1} Y \leq 0 .
\]

Therefore, \(\mathcal{K}_{M, S} \subseteq \{ \bar{X} : s_{11}^{-1} Y \leq 0, Z - s_{21} s_{11}^{-1} Y > 0 \} \).

(1i) To prove the opposite inclusion, suppose \(\bar{X}\) is such that \(s_{11}^{-1} Y \leq 0, Z - s_{21} s_{11}^{-1} Y > 0\). We show that
\[
(4.2) \quad \frac{\uhat}{\mu} = \hat{\mu} = \begin{pmatrix} 0 \\ Z - s_{21} s_{11}^{-1} Y \end{pmatrix} ,
\]

for this implies \(\bar{X} \in \mathcal{K}_{M, S}\). Note that
\[
(4.3) \quad (\hat{\mu} - \bar{X}) = -\begin{pmatrix} Y \\ s_{21} s_{11}^{-1} Y \end{pmatrix} = -\begin{pmatrix} I \\ s_{21} s_{11}^{-1} \end{pmatrix} Y .
\]

Therefore,
\[ s^{-1}(\mu^* - \bar{x}) = -\frac{(s^{11} + s^{12}S_{21}s^{-1})Y}{(s^{21} + s^{22}S_{21}s^{-1})} \]

\[ = -\left( \begin{array}{c} s^{-1}Y \\ 0 \end{array} \right) \geq 0. \]

Thus (2.6) is satisfied by \( \mu^* \). (2.7) follows immediately and so Lemma 2.1 implies \( \mu_S = \mu^* \).

We now need the following identity to express \( U \) in terms of \( Y \) and \( Z \) when \( \bar{x} \in \mathcal{K}_{M,S} \):

\[(4.4) \quad (a'b')S^{-1}_{11} = (b' - S_{21}S^{-1}_{11}a)'S^{-1}_{11}S_{22}a + a'S^{-1}_{11}a, \]

where \( a \) is \((p-k) \times 1\), \( b \) is \( k \times 1\), and \( S_{22} = S_{22} - S_{21}S^{-1}_{11}S_{12} \).

From (4.2), (4.3), and (4.4) we see that \( \bar{x} \in \mathcal{K}_{M,S} \) implies

\[ U = \frac{N[Z + S_{21}S^{-1}_{11}Y]'S^{-1}_{11}S_{21}S^{-1}_{11}[Z + S_{21}S^{-1}_{11}Y]}{1 + NY'S^{-1}_{11}Y}. \]

Let

\[ \alpha = \frac{N[Z - S_{21}S^{-1}_{11}Y]'S^{-1}_{22}S_{22}S^{-1}_{11}[Z - S_{21}S^{-1}_{11}Y]}{N[Z - S_{21}S^{-1}_{11}Y]'S^{-1}_{22}S_{22}S^{-1}_{11}[Z - S_{21}S^{-1}_{11}Y]}, \]

\[ \gamma = N^{1/2} \frac{Z - S_{21}S^{-1}_{11}Y}{(1 + NY'S^{-1}_{11}Y)^{1/2}}. \]

Then by Lemma 4.1 and the above expression for \( U \),

\[(4.5) \quad P[U \geq c, \bar{x} \in \mathcal{K}_{M,S}] = P[\alpha(\gamma'S^{-1}_{22,1}\gamma) \geq c, \gamma > 0, S^{-1}_{11}Y \leq 0] . \]

To evaluate this last probability we need the following standard results concerning the joint distribution of \( (\bar{x}, S) \), which may be found, for example, in Stein (1966):
(4.6) Under $H$ (or $K$), $\alpha$ is independent of 
$(\gamma, S_{11}, Y)$ and $1/\alpha \sim \chi^2_{N-p+1}$.

(4.7) Under $H$, $\gamma$ is independent of $(S_{11}, Y)$ and 
$\gamma \sim \mathcal{N}(0, \Sigma_{22,1})$, so $\gamma' \Sigma_{22,1}^{-1} \gamma \sim \chi^2_k$.

From (4.6) and (4.7) it follows that under $H$, $\alpha$, $\gamma$, $S_{11}$, and $Y$ are 
mutually independent. Therefore,

(4.8) $P[\alpha(\gamma' \Sigma_{22,1}^{-1} \gamma) > c, \gamma > 0, S_{11}^{-1} Y \leq 0]$

$= P[\alpha(\gamma' \Sigma_{22,1}^{-1} \gamma) > c, \gamma > 0] P[S_{11}^{-1} Y \leq 0]$

and

(4.9) $P[\chi_{M, S}^2] = P[\gamma > 0, S_{11}^{-1} Y \leq 0] = P[\gamma > 0] P[S_{11}^{-1} Y \leq 0]$.

Now consider the term $P[\alpha(\gamma' \Sigma_{22,1}^{-1} \gamma) > c, \gamma > 0]$. The vector 
$\Sigma_{22,1}^{-1/2} \gamma \sim \mathcal{N}(0, I)$ is orthogonally invariant, so Lemma 2.2 implies that 
the events $\{\gamma' \Sigma_{22,1}^{-1} \gamma > d\}$ and $\{\gamma > 0\}$ are independent. Therefore,

(4.10) $P[\alpha(\gamma' \Sigma_{22,1}^{-1} \gamma) > c, \gamma > c]$

$= E[P[\alpha(\gamma' \Sigma_{22,1}^{-1} \gamma) > c, \gamma > 0 \mid \alpha]]$

$= E[P[\alpha(\gamma' \Sigma_{22,1}^{-1} \gamma) > c \mid \alpha] P[\gamma > 0]]$

$= P[\alpha(\gamma' \Sigma_{22,1}^{-1} \gamma) > c] P[\gamma > 0]$

$= P[\chi^2_k / \chi^2_{N-p+1} > c] P[\gamma > 0]$,

where $\chi^2_k$ and $\chi^2_{N-p+1}$ denote independent chi-square variates with $k$
and $N-p+1$ degrees of freedom, respectively. Combining (4.5), (4.8), (4.9), and (4.10) we obtain
\[(4.11) \ P[U \geq c, \bar{X} \in \mathcal{K}_{M,S}] = \ P[\chi_k^2 / \chi_{N-p+1}^2 \geq c] \ P[\gamma > 0] \ P[S_{11}^{-1} y \leq 0] \]

\[= \ P[\chi_k^2 / \chi_{N-p+1}^2 \geq c] \ P[\bar{X} \in \mathcal{K}_{M,S}].\]

Note that the first factor depends on \( M \) only through \( k \), the number of elements of \( M \). Let

\[K(\hat{\mu}_S) = \text{the number of positive components of } \hat{\mu}_S,\]

\[K(\hat{\mu}_\Sigma) = \text{the number of positive components of } \hat{\mu}_\Sigma,\]

\[w'(p,k,\Sigma) = P[K(\hat{\mu}_S) = k],\]

\[w(p,k,\Sigma) = P[K(\hat{\mu}_\Sigma) = k].\]

Then \( \{K(\hat{\mu}_S) = k\} = \bigcup_{\#(M)=k} \mathcal{K}_{M,S} \) where the union extends over all \( M \) having exactly \( k \) elements. Summing \((4.11)\) over all such \( M \) we have

\[(4.12) \ P[U \geq c, K(\hat{\mu}_S) = k] = \ P[\chi_k^2 / \chi_{N-p+1}^2 \geq c] \ P[K(\hat{\mu}_S) = k] \]

\[= \ P[\chi_k^2 / \chi_{N-p+1}^2 \geq c] \ w'(p,k,\Sigma).\]

Thus, we have proved

**Theorem 4.2.** The distribution under \( H \) of the LRT statistic \( U \) in \((3.4)\) is as follows:

\[P[U \geq c] = \sum_M P[\chi_k^2 / \chi_{N-p+1}^2 \geq c] \ P[\bar{X} \in \mathcal{K}_{M,S}].\]

\[= \sum_{k=1}^{p} P[\chi_k^2 / \chi_{N-p+1}^2 \geq c] \ w'(p,k,\Sigma).\]

**Remark.** Notice that \( P[U = 0] = w'(p,0,\Sigma) > 0.\)

We now give simplified expressions for the weights \( w'(p,k,\Sigma), \) and at the same time show the relation between \( w'(p,k,\Sigma) \) and \( w(p,k,\Sigma). \)
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\[(4.18)\quad w'(p,k,\Sigma) = w(p,k,\Sigma) = \sum_{\#(M)=k} \psi(\Sigma_m^{-1/2}) \psi(\Sigma_1^{1/2}),\]

\[P[U \geq c, K(\hat{\mu}_S) = k] = P[\chi^2_k/\chi^2_{N-p+1} \geq c] \quad P[K(\hat{\mu}_S) = k] = P[\chi^2_k/\chi^2_{N-p+1} \geq c] \quad w(p,k,\Sigma).\]

Therefore, Theorem 4.2 can be restated as follows:

**Theorem 4.2'.** The distribution under H of the LRT statistic U in (3.4) is as follows:

\[P[U \geq c] = \sum_M P[\chi^2_k/\chi^2_{N-p+1} \geq c] \quad P[\bar{X} \in \mathcal{K}_M, \Sigma] = \sum_{k=1}^{p} P[\chi^2_k/\chi^2_{N-p+1}] \quad \omega(p,k,\Sigma),\]

where the weights \(w(p,k,\Sigma)\) are the weights occurring in (2.10) for the case \(\Sigma\) known.
References


