Speaker: Sumit Mukherjee, Stanford University

Title: Persistence probabilities for Gaussian polynomials

Abstract:
Consider the random polynomial $Q_n(x) := \sum_{i=0}^{n} \xi_i x^i$, where $\xi_i$’s are mutually independent with $\xi_i \sim N(0, i^\alpha)$, $\alpha \in \mathbb{R}$, and let $N_n$ denote the number of real roots of $Q_n(x)$ in $\mathbb{R}$. The special case $\alpha = 0$ (which corresponds to i.i.d. coefficients $\xi_i$) has been an object of much interest in probability, starting with Kac who derived an explicit formula for $\mathbb{E}N_n$ and showed that $\mathbb{E}N_n \sim \frac{2}{\pi} \log n$.

The main question addressed by this talk is the following: “How small is $\mathbb{P}(N_n = 0)$, the probability that there are no real roots of $Q_n(x)$?” It was shown in 2002 by Dembo et al. that in case $\alpha = 0$, the probability above decays polynomially in $n$, and characterized the decay exponent. Physicists conjectured that polynomial decay holds for all $\alpha > -1$. We show that this is indeed true, and give a characterization of the limit. Further, our argument is robust and gives a general criteria for continuity of persistence coefficients for Gaussian processes. Using this technique we are also able to compute persistence coefficients for variance of the form $\frac{1}{i}$ or $\binom{n}{i}$, which are other common random polynomials in Physics literature.

This is joint work with Professor Amir Dembo, Stanford University.