Title: Permutation Tests 101

Abstract:

Given independent samples from $P$ and $Q$, two-sample permutation tests allow one to construct exact level tests when the null hypothesis is $P = Q$. On the other hand, when comparing or testing particular parameters $\theta$ of $P$ and $Q$, such as their means or medians, permutation tests need not be level $\alpha$, or even approximately level $\alpha$ in large samples. Under very weak assumptions for comparing estimators, we provide a general test procedure whereby the asymptotic validity of the permutation test holds while retaining the exact rejection probability $\alpha$ in finite samples when the underlying distributions are identical. The ideas are broadly applicable and generalized to the Wilcoxon test, and to the $k$-sample problem of comparing general parameters, whereby a permutation test is constructed which is exact level $\alpha$ under the hypothesis of identical distributions, but has asymptotic rejection probability $\alpha$ under the more general null hypothesis of equality of parameters. A quite general theory is possible based on a coupling construction, as well as a key contingency argument for the multinomial and multivariate hypergeometric distributions. Time permitting, the results will be extended to multivariate settings and multiple testing.