Stanford University
Departments of Mathematics and Statistics

Probability Seminar

4:15pm, Monday, October 22, 2012
Sequoia Hall Room 200
Cookies served at 3:45pm, 1st floor Lounge.

Speaker: Yashodhan Kanoria, Microsoft Research New England

Title: The Set of Solutions of Random XORSAT Formulae

Abstract:
The XOR-satisfiability (XORSAT) problem requires finding an assignment of $n$ Boolean variables that satisfies $m$ exclusive OR (XOR) clauses, where each clause constrains a subset of the variables. We consider random XORSAT instances, drawn uniformly at random from the ensemble of formulae containing $n$ variables and $m$ clauses of size $k$ each. This model presents several structural similarities to other ensembles of constraint satisfaction problems, such as $k$-satisfiability ($k$-SAT). For many of these ensembles, as the number of constraints per variable grows, the set of solutions shatters into an exponential number of well-separated components. This phenomenon appears to be related to the difficulty of solving random instances of such problems.

We prove a complete characterization of this clustering phase transition for random $k$-XORSAT. In particular we prove that the clustering threshold is sharp and determine its exact location. We prove that the set of solutions has large conductance below this threshold and that each of the clusters has large conductance above the same threshold. Our proof constructs a very sparse basis for the set of solutions (or the subset within a cluster). This construction is achieved through a low complexity iterative algorithm.

This is based on joint work with Morteza Ibrahimi, Matt Kraning and Andrea Montanari.