STATISTICAL APPLICATIONS IN ARMS CONTROL INSPECTION

BY

HERBERT SOLOMON

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STATISTICAL APPLICATIONS IN ARMS CONTROL INSPECTION*

By

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For the past several years, and on a rather infrequent basis, I have looked into some arms control inspection problems where the development of statistical procedures seemed natural and could be useful. This has not been done for government agencies or on contracts with such agencies except for a two-day work panel in 1963. Therefore, what I have to offer this audience may be completely non-realistic or banal.

My career in this subject began about 1957 in what might be called a typical way for those engaged in the practice of statistics. At that time I was at Columbia, and a colleague, Seymour Melman, phoned to ask if I would prepare a memo on some possible uses of statistical methodology in inspection problems arising from disarmament policies. At that time, Melman was conducting a research program through the Institute of War and Peace Studies at Columbia on the possibility and feasibility of inspection procedures which would turn arms control agreements between nations into pragmatic pacts. It is needless to say that this was a rather new subject at that time to all who participated in it.

The memo I prepared turned into a brief chapter and appeared in a volume published in 1958 together with 20 or so other contributions by engineers, physicists, economists, sociologists, mathematicians,

* This is a written version of an invited talk given at the Annual Meeting of the Operations Research Society of America (Western Section), held in Las Vegas, Nevada, Sept. 29-30, 1965.
psychologists, biologists, chemists and military officials. This volume was edited by Melman and was entitled 'Sampling Inspection for Disarmament' (Columbia University Press). It was prepared quickly and yet, in retrospect, it was possibly the first comprehensive study framed in an operations research setting on a topic which had received much political and public discussion after Hiroshima and now obviously will be with us for some time. Many of the contributors who were employing the tools and resources of their own discipline in a brand new problem area might get a kick out of re-reading their output of some 7-8 years ago. When I was invited to participate in this session, I summoned my courage and looked at the very brief chapter that resulted from my handiwork.

In the technological framework of that era inspection meant an actual visitation to metal working plants, biological laboratories, pharmaceutical plants, and similar units or actual confrontation with scientific personnel records, budget and audit records, and similar records of a nation participating in an arms control pact. Inspection by monitoring devices - electronic, acoustical, or photographic - physically located in one country to gather information on events in another country; or inspection by air reconnaissance; or inspection by satellites did not receive too much attention.

Accordingly my chapter discussed sampling strategies for visiting teams and employed what were traditional sample survey techniques, mainly stratified sampling. It also included in a very scanty way some questions of economic costs and the possible uses of available intelligence to sharpen sampling designs. This analysis which today might be labelled a primitive cost-effectiveness study indicated the feasibility of this type of inspection in terms of the manpower and economic resources necessary for such inspection undertakings.
To demonstrate the difference in climate a few short years can make, it is instructive to quote from the Foreword to Melman's volume which was written by Professor Wm. Fox who served then as Director of the Columbia University Institute of War and Peace Studies. I quote:

"Agreements between governments have so far been hindered by an inability to discover a "disarmament package" which did not seem to the other side to favor the side proposing it. Neither the U. S. nor the Soviet Union has been prepared to enter into an agreement which might leave it at the mercy of the other if the agreement were violated or if the system broke down... (he continues) New vistas for negotiation and agreement would be opened by a technically well-founded judgment that it would be possible to install on this planet a disaster-proof alarm system to detect and identify a clandestine violator of a disarmament agreement. To be proof against such a disaster, it would have to give to the law-abiding signatories to an arms control agreement warning of a violation in time for effective counter measures."

An Arms Control Model:

There are two areas of analysis that follow from this portion of a Foreword written in what seems like the distant past which I would now like to present through discussion of more recent work. The first portion of the quote (namely, reference to discovery of a "disarmament package") is exemplified by an article in ORSA, July 1964 written by Thomas Saaty, who is with the U. S. Arms Control and Disarmament Agency. This paper is entitled "A Model for the Control of Arms." In the first part of his paper he goes into a mathematization to describe different levels of arms available to two competitive opponents, named X and Y.
These two opponents each select a set of rules to be applied to an initial state of arms to produce a new state. It is assumed that the initial state of arms to which the rules are applied will be considered as an equilibrium state by both sides, where equilibrium takes into account military, political, economic, and other factors. The same rule or different rules may be applied to the new state to obtain a third state, etc. Opponent X's total scheme of arms reduction will produce a set of states any one of which need not be acceptable to opponent Y and vice versa. The objective is to find those states on which there can be agreement by both sides and then establish rules for reducing arms in those states. We have seen this, in a way, by the 1963 pack which banned nuclear detonations except those occurring in underground testing. Apparently the hindrance noted by Prof. Fox in 1958 was over-come by technological advances in detection of nuclear detonations and we shall return to this point.

Suppose we now look into Saaty's model a bit. Let the set \( \sum \) consist of the finite number of states where each state represents a level of arms available to X and Y. The elements of \( E_j (j=1, 2, \ldots, p) \) of \( \sum \) are called states and represented by vectors,

\[
E_j = \left( a_{1j}, \ldots, a_{nj}; b_{1j}, \ldots, b_{nj} \right)
\]

where \( a_{kj} \) denotes the number of weapons (guns, men, missiles, amount of information, cost of getting information) of type \( k \) available to X at step \( j \) of the disarmament process; similar definition for \( b_{jk} \) for Y. An equilibrium state is one which is admissible for both sides. A natural criterion for selecting admissible states is for X to take the value

\[
a_{kj} = \alpha_{jk} b_{kj}
\]
where $\alpha_{kj}$ is the compensating factor. The value of $\alpha_{kj}$ must
be measured in the basic unit of both weapons.

Define a state $E_j$ to be admissible as a candidate for the set $\sum_x$
of $X$'s states if $\|\alpha_j\|$ called the norm of $\alpha_j = (\alpha_{1j}, \alpha_{2j}, \ldots, \alpha_{nj})$ -
the vector of compensating factors - is not less than a specified value.
A useful norm to take for $\alpha_j$ is

$$\|\alpha_j\| = \sum_{k=1}^{n} w_k \alpha_{jk}$$

where $w_k$ measures the importance of weapon type $k$.

Note for example that the state $(0, \ldots, 0; 1, \ldots, 1)$ is admissible
to $Y$ but not to $X$ and hence it is in $\sum_x$ but not in $\sum_y$; likewise
$(1, \ldots, 1; 0, \ldots, 0)$ is in $\sum_x$ but not in $\sum_y$. It is easy to assume such
states are admissible since one side will have zero arms. The set of
equilibrium states admissible to both $X$ and $Y$ is $\sum = \sum_x \cap \sum_y$,
that is, it is the part common to both. Here $\sum = \sum_x \cap \sum_y$ is a null set.

The problem of finding transition rules to achieve a sequence of
equilibrium states is a large order but it is a necessary piece to the
development. It is only in equilibrium states that reduction of arms is
considered possible.

Once found, the next problem is finding a method of applying these
rules to obtain all those states that fall on a path of arms reduction
from a given initial state. If there is no such path, the rules are inad-
quate and must be altered.

Saaty illustrates his development through a modification of a well-
known puzzle.

Problem: Three missionaries and three cannibals come to
the left bank of a river which they need to cross in a boat
that can take only two people at a time. The boat must always
be considered to be on one side or the other side of the river.
The crossing must be made in such a way that on neither side
can there ever be more cannibals than missionaries (including those in the boat), since even after years of religious indoctrination the cannibals retain their old habits. How can six men be transferred across — assuming that they all know how to row?

In order to take as little time as possible and still illustrate the idea of equilibrium states, Saaty considers a modified problem.

Solve the problem of ferrying across from the left bank to the right bank of a river two missionaries and two cannibals all of whom can row. We start by listing all admissible states. Let \( X \) be the set of missionaries and let \( Y \) be the set of cannibals.

A state is represented by a pair of numbers the first of which gives the number of missionaries and the second gives the number of cannibals. The states of \( \sum_X \) are \((2, 2), (2, 1), (2, 0), (1, 1), (0, 1), (0, 2), (0, 0)\). Thus the missionaries allow all those states in which their number is equal to or more than the number of cannibals. On the other hand \( \sum_Y \) is \((2, 2), (2, 1), (2, 0), (1, 2), (0, 2), (1, 1), (1, 0), (0, 1), (0, 0)\). The states that are common to both give \( \sum \).

They are admissible to both sides. The only two states in \( \sum_Y \) that are not in \( \sum_X \) are \((1, 0)\) and \((1, 2)\). The reason for the second state is clear, since there are more cannibals than missionaries. The state \((1, 0)\) implies that on the opposite side of the river we have the state \((1, 2)\) in which there are more cannibals than missionaries. Thus we have the states of \( \sum \) on the left bank and their complements on the right bank:

<table>
<thead>
<tr>
<th>Left Bank</th>
<th>Right Bank</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_1 = (2, 2) )</td>
<td>( (0, 0) )</td>
</tr>
<tr>
<td>( E_2 = (2, 1) )</td>
<td>( (0, 1) )</td>
</tr>
<tr>
<td>( E_3 = (2, 0) )</td>
<td>( (0, 2) )</td>
</tr>
<tr>
<td>( E_4 = (1, 1) )</td>
<td>( (1, 1) )</td>
</tr>
<tr>
<td>( E_5 = (0, 2) )</td>
<td>( (2, 0) )</td>
</tr>
<tr>
<td>( E_6 = (0, 1) )</td>
<td>( (2, 1) )</td>
</tr>
<tr>
<td>( E_7 = (0, 0) )</td>
<td>( (2, 2) )</td>
</tr>
</tbody>
</table>
Let us associate with each state $E_j$ belonging to $\sum$ a vertex $v_j$ ($j = 1, 2, \ldots, r$) of a linear graph. If the departure of the boat always leaves behind a state that is in $\sum$, we join the points corresponding to the states before and after the departure. With this graph we can associate a vertex matrix whose elements are zero or one depending on whether transitions from one state on the left bank to another on the left bank are possible. The transitions of course are effected by the departure of the boat. We have the vertex matrix describing whether it is possible to go from the state represented by the vertex on the left to another state represented by a vertex on top.

\[
V = \begin{bmatrix}
  v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \\
  v_1 & 0 & 1 & 1 & 1 & 0 & 0 \\
  v_2 & 0 & 0 & 1 & 1 & 0 & 1 \\
  v_3 & 0 & 0 & 0 & 0 & 0 & 1 \\
  v_4 & 0 & 0 & 0 & 0 & 1 & 1 \\
  v_5 & 0 & 0 & 0 & 0 & 1 & 1 \\
  v_6 & 0 & 0 & 0 & 0 & 0 & 1 \\
  v_7 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

On the right bank of the river one would have an identical set of states that are essentially the complements to those on the left bank. Their matrix would be $V'$, the transpose of the above matrix. Thus the matrix of transition after one round trip must be $VV'$. In general for the transition matrix of $n$ round trips we have $(VV')^n$ and to move to the right bank after $n$ round trips we have $(VV')^n V$.

Thus the problem is to find the number of round trips $n$ such that the element in the $v_1, v_7$ position of this product is one, i.e., we have the transition $(2, 2) \rightarrow (0, 0)$ on the left bank - or equivalently the group has moved to the right bank.

It turns out that $n = 2$ for this problem, i.e., two round trips and
one last trip forward, for a unit element appears for the first time for the \( v_1 \), \( v_7 \) position in \((VV')^2 \cdot V\).

The problem now is to read the solution from the matrices. Where could the 1 in \( v_1 \), \( v_7 \) have come from? There is a unit element in the (1, 4) position of \((VV')^2\) and a unit element in the (4, 7) position of \( V \). Thus the last transition is \( v_4 \rightarrow v_7 \), and working along we end up as follows

\[ v_1 \rightarrow v_3', v_3 \rightarrow v_2', v_2 \rightarrow v_6', v_6 \rightarrow v_4', v_4 \rightarrow v_7 \]

or simply

\[ v_1', v_3', v_2', v_6', v_4', v_7' \]

This is a novel representation of an old parlor puzzle which was proposed and developed by Benjamin Schwartz in Math. Magazine, Vol. 34, 1961 and is used by Saaty to bring out his ideas of equilibrium states. In the usual Tartaglia or transportation problem puzzle there are 3 missionaries and 3 cannibals where all missionaries and only one cannibal can row. There are 16 possible states to this problem \((v_1', v_2', \ldots, v_{16}')\) and the same procedure can be employed to arrive at the solution. In this case, six round trips plus one additional final crossing provides the solution.

To determine whether a solution exists for a given problem, Schwartz argues as follows: Consider

\[ (VV')^n \leq (VV')^{n'} \quad \text{and} \quad n < n'. \]

To every unit element in the \( i, j \) position in the matrix on the left there corresponds a unit element in the \( i, j \) position of the matrix on the right i.e., if the vertex \( v_i \) can be reached in \( n \) round trips from \( v_j \) then it can also be reached in \( n' \) round trips. This is obviously so since \((n'-n)\) round trips can be made with the same passenger list. Thus the sequence \((VV')^n \) \((n = 1, 2, \ldots)\) is monotone increasing and bounded above with upper bound, say \( Q \) which is
obtained from

\[ Q(VV^t) = Q. \]

If the \( i, j \) element of \( Q \) is 1, the problem is solvable - otherwise it is not. For instance - the four missionary and four cannibal problem is not solvable.

Saaty's paper contains other areas of discussion. The particular mathematical model just presented struck me as particularly pertinent because of the remarks I made earlier in connection with Professor Fox's foreword. It also demonstrates how even the simplest situations lead to difficult and perhaps intractable mathematical developments.

In my remarks on Saaty's paper the techniques presented are more of an algebraic than of a statistical nature. One runs into this in model building and conceptualizations; even for statistical universes. However, it is easy to lapse into statistical questions even in terms of the model itself; for example, when either \( a_{kj} \) or \( b_{kj} \) is not known how can one estimate the compensating factor \( \alpha_{kj} \). Knowledge of \( \alpha_{kj} \) is essential for determining admissible states. An alternative approach is to define \( \sum \) in terms of states that are admissible with prescribed probabilities.

**Detection Models:**

There are a number of papers in the last three years or so that deal with detection in the format of probabilistic models and which have varying degrees of association with arms control and disarmament. On one end of this spectrum there is a paper by Mel Dresher of RAND presented at a SIAM meeting about two years ago which discusses a sampling inspection problem in arms control agreements from a game-theoretic point of view - where the sampling procedure to monitor disarmament agreements can be formulated as a zero-sum two-persons games. For example, the sampling problem of selecting a fixed number of items, say \( m \), for close inspection out of a series of \( n \) items could arise...
when a ban on underground nuclear tests is to be verified by on-site inspection of a limited number of those seismic shocks that cannot be identified by seismology to be earthquakes. We will return to this particular problem in a more central way soon. In his paper Drescher looks into such indexes as the expected payoff to an inspector to make $m$ inspections out of $n$ consecutive events during some time period; also the optimal strategy of an inspector or a violator if there are $n$ events and the probability of detecting a violation is some value $0 < p < .5$. That is, we determine the probability of inspecting the next event if the inspector has $m$ inspections left and there are $n$ periods to go - likewise the probability of scheduling a violation during the $n$th event if the inspector has $m$ inspections left and if no violation has yet occurred. Also relevant here, is a paper by Blackwell and Hodges in Annals Math. Statist., 1957 on control of selection bias, i.e., if each inspection is selected independently with a probability $p$ until $m$ inspections are made - what is the best sampling design knowing that each inspection can introduce a bias, that is a violation, in the next event.

Going along the spectrum, there are recent papers in ORSA on such topics as "Surveillance of a Region by Detection and Tracking Operations" (Jas. Dobbie) and "Minimax Detection Station Placement" (Richard Smallwood) which relate pretty directly to arms control policy, especially the latter paper.

There is a class of papers initiated by Shirayev in Russian journals and continued by others here and abroad on detection of the arrival of a "disorder" in an observed stochastic process as quickly as possible subject to a limitation on the number of false alarms. This may have less direct connection but could be important.

For the balance of this talk, I would like to present some statistical questions which could arise from underground nuclear testing since
this is permitted under the 1963 pact and thus has a strong aura of realism about it.

Here, inspection teams are not permitted direct access yet both the U. S. and the USSR, and now others, would like information when such testing occurs since it may not be publicized. One may assume that intelligence of various kinds and from various sources is gathered. A major question is whether a seismic shock recorded in some situations is due to an earthquake or an underground nuclear detonation. The fact that this is the only nuclear testing permitted suggests that both sides feel that monitoring is a simple matter or both sides feel that monitoring is not easily accomplished. Here one can resort to thinking in Saaty's model for some possible clarification.

Now for the important question of determination of seismic event location since this in turn will establish a stronger or weaker argument that one country has performed unpublicized underground nuclear testing. We can view problems here as falling into one of each of two categories and this classification comes about naturally.

In one situation data is made available from stations reporting a seismic event and questions of prediction of, and confidence regions for, the coordinates of the actual location of the event are to be resolved on the basis of the sample data. The second situation arises from the consideration of the probability that a geographic region whose centroid is random and is governed by a bivariate distribution will cover the true location of the seismic event. In what follows, some comments and possibly a resolution of some aspects of these problems are presented.

1. Estimation and Prediction: Confidence Regions

The confidence region statement for the coordinates of the true epicenter of a seismic event (assuming focal depth is zero) is an inference based on data collected at seismic listing stations. Assume \( n \) stations
obtain a record of a seismic event. At least three stations are required to estimate the epicenter uniquely. This is typical for getting a "fix" on a position. Usually the number of stations is much larger in number than three (order of 30-50) but in some cases the number may be quite small. The coordinates of the epicenter are estimated by least squares and are derived from travel time readings obtained at the listening stations.

For example, let

\[ \tau_o = \text{true origin time} \]
\[ t_i = \text{the observed arrival time at the } i^{th} \text{ station} \]
\[ \tau_i = \text{the true arrival time at the } i^{th} \text{ station} \]
\[ \tau_i = \tau_o + f_i (\theta, \varphi, h) \]
\[ f_i = \text{a known function for each } i \text{ and is the true travel time from the event to the } i^{th} \text{ station} \]

and \( \theta, \varphi, h \) are location parameters of the hypocenter. We assume \( h \) is zero so that only \( \theta, \varphi \); the coordinates of the epicenter, are to be estimated.

Least squares solutions for \( \tau_o, \theta, \varphi \) are obtained if

\[ S = \sum_{i=1}^{n} (t_i - \tau_i)^2 \]

is minimized. These are \( \hat{\tau}_o, \hat{\theta}, \hat{\varphi} \) (and consequently \( \hat{\tau}_i, \hat{f}_i \)). Usually the assumption is made that the \( t_i \) are normally and independently distributed with mean \( \tau_i \) and common variance \( \sigma^2 \). Then these estimates are equivalent to maximum likelihood estimates.

For a seismic event we have an observation, \( t_i \), from each of \( n \) stations. Each \( t_i \) has a conditional expected value \( \tau_i \) which depends on \( \tau_o, \theta, \varphi \). This provides the least squares model which leads to the \( n \) equations in which the three parameters which serve as the unknowns are implicit. Actually when a solution is obtained, corrections resulting from the physics of time and distance for waves on the surface of the earth are applied to the values obtained for the epicenter and \( \tau_o \) from the least squares equations. This is treated as a new input and the entire process of computing residuals and finding changes in the epicenter and \( \tau_o \) is repeated until the changes are less
than some predetermined value. Apparently this process converges rapidly (very few iterations are required) and the epicenter to which the process converges satisfies the least squares criterion for station residuals.

This solution may be programmed so that the final output from a computer consists of the following estimates: coordinates of epicenter; origin time; mean and standard deviation of the residuals, standard error of estimate of the location. It is this last index, namely \( \hat{S} \), where
\[
\hat{S}^2 = \frac{1}{n-3} \sum_{i=1}^{n} (\hat{t}_i - \hat{\theta}_i)^2
\]
which is important in the construction of confidence regions for the epicenter coordinates \((\theta, \varphi)\). Assume there is no bias so that it estimates the unknown but assumed equal variance of measurement error for each station. (If equal variance cannot be assumed, the analysis becomes more intractable; zero correlation between stations is usually assumed.)

If the number of stations, \( n \), is quite large then
\[
\frac{(n-3)\hat{S}^2}{\sigma^2} = \frac{1}{\sigma^2} \sum_{i=1}^{n} (\hat{t}_i - \hat{\theta}_i)^2
\]
can be approximated by a chi-square distribution with \((n-3)\) degrees of freedom. (Recall that \( \hat{t}_i \) is a function of \( \hat{\theta}_i, \hat{\varphi}_i \).) If a partitioning of this total sum of squares into two sums of squares can be achieved so that one involves the \( f_i(\theta, \varphi) \) only, and the other involves \( \hat{\theta}_i \) and \( \hat{\varphi}_i \); and if Cochran's Theorem on quadratic forms and independent chi-square distributions holds, then the ratio of these two sums of squares (each divided by an appropriate constant) will have an F distribution. For a specified confident coefficient, say 95%, an appropriate value can be obtained from the F tables and the ratio set equal to this value. This yields implicitly a 95% confidence region for \( \theta, \varphi \) which can be obtained from sample data.

This approach has additional analytic difficulties which I now stress. For large \( n \), one can go along with the chi-square approximation but the
magnitude of \( n \) such that one can tolerate the approximation is an open and nagging question. If \( n \) is small, the chi-square approximation may prove to be very poor for the total sum of squares. Thus some idea of the exact distribution for small \( n \) is of importance to handle that case and also to give information on how the error due to the approximation decreases as \( n \) increases.

Since it is a reasonable guess that the exact distribution is the distribution of some linear function of chi-squares and thus may be approximated by a chi-square with non-integer degrees of freedom, it may be profitable to fit chi-square distributions to data collected by simulation on a computer. This should be done on an extensive basis say for 6, 8, 10, 12, 14, 16, 18, 20, 30, 40, 50 stations. In other words - find empirical distributions of \( \hat{s}^2 \) from simulated data and examine these for which chi-square distributions fit best, then see if the value of the degrees of freedom for best fit checks with some conjectures about the chi-square approximations.

2. Coverage

Here we have a problem which has received some attention in the literature. In our situation there is a geographical region \( R \) whose centroid \( \hat{\theta}, \hat{\varphi} \) is distributed around a fixed point \( \theta, \varphi \) the true location of the seismic event on the earth's surface. The random centroid is the estimated epicenter and the deviations between \( \theta, \varphi \) and \( \hat{\theta}, \hat{\varphi} \) are assumed to follow a bivariate normal distribution with known covariance matrix. What is of interest is the probability that the region \( R \) will cover the point \( \theta, \varphi \) or equivalently, the dimensions of the region \( R \) are sought so that the point \( \theta, \varphi \) will be covered at a specified probability level.

Suppose the elements of the covariance matrix are \( \sigma^2_{\theta}, \sigma^2_{\varphi} \) and the covariance \( \sigma_{\theta\varphi} = 0 \). If the latter is not so, then a transformation can always be found to bring us to this setting and to accomplish what
follows. If we let
\[ \sigma^2 = \sigma\theta^2 + \sigma\varphi^2, \quad a_1 = \sigma\theta^2 / \sigma^2, \quad a_2 = \sigma\varphi^2 / \sigma^2, \quad t = R^2 / \sigma^2 \]

then we may write
\[ P[a_1 \chi_1^2 + a_2 \chi_2^2 \leq t] = p, \]

where \( R \) is the radius of the circular region \( R \) which covers the point \( \theta, \varphi \) with probability \( p \). Likewise, if we set \( p = .95 \), we can find the magnitude of the radius of the circular region \( R \) which accomplishes this. This analysis and accompanying tables can be found in --


More extensive tables can be found in --


If \( \sigma\theta^2 = \sigma\varphi^2 \), the work is simplified.

When the region \( R \) is an ellipse, square, or rectangle, some answers can be obtained. For the case of an ellipse, the following may be useful --


For cases where \( R \) is a square or a rectangle, these regions can be transformed into probi-equivalent circles and the previous approach applied.
Statistical Applications in Arms Control Inspection

Technical Report

Solomon, Herbert

December 20, 1965

15

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Logistics and Mathematical Sciences Branch
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Washington, D.C. 20360

Several statistical techniques appropriate for problems arising in arms control inspection are presented and discussed. A major portion of the paper is devoted to methodology for detection of seismic shocks. Also discussed is an interesting model developed by Satty by which different levels of arms between opponents can be described, thus permitting a mathematization of transition rules to obtain equilibrium states and, as a consequence, arms reduction. Some work of others is given brief attention.
Arms Control Inspection
Detection of Seismic Shocks

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