OPTIMAL PURSUIT STRATEGIES FOR THE LION

BY

JAMES FLYNN

TECHNICAL REPORT NO. 174
APRIL 22, 1971

THIS RESEARCH WAS SPONSORED BY THE ARMY RESEARCH OFFICE
OFFICE OF NAVAL RESEARCH, AND AIR FORCE OFFICE OF
SCIENTIFIC RESEARCH BY CONTRACT NO.
N00014-67-A-0112-0053 (NR-042-267)

DEPARTMENT OF STATISTICS
STANFORD UNIVERSITY
STANFORD, CALIFORNIA
OPTIMAL PURSUIT STRATEGIES FOR THE LION

by

JAMES FLYNN

TECHNICAL REPORT NO. 174
April 22, 1971

PREPARED UNDER CONTRACT N00014-67-A-0112-0053
NR-042-267
OFFICE OF NAVAL RESEARCH

Herbert Solomon, Project Director

Reproduction in Whole or in Part is Permitted for any Purpose of the United States Government

DEPARTMENT OF STATISTICS
STANFORD UNIVERSITY
STANFORD, CALIFORNIA
OPTIMAL PURSUIT STRATEGIES FOR THE LION

by

James Flynn

In [1] and [5] the following problem is considered: A lion (L) and a man (M) in a circular arena have equal speeds. Can the lion catch the man in a finite time? Besicovitch [5] shows that he cannot while Croft [1] shows that he can if we require that the curvature of the man's path exist and be uniformly bounded. In [3] we consider a modification of the above problem suggested by Blackwell. If the man's speed is greater than that of the lion, then how close can the lion get to the man and can he achieve that distance in a finite time?

This report is a continuation of [3]. There we formulate the problem as a differential game, show that the game has a value $\rho^*$ and construct upper and lower bounds on $\rho^*$ and an optimal evasion strategy for the man. Here we determine optimal pursuit strategies for the lion. We assume that the reader is familiar with the notation, definitions and results of [3].

Our main result is that the following pursuit strategy is optimal when $\omega > 1$ where $\omega$ represents the ratio of the man's speed to the lion's: Go to the center $O$. Then employ the radial strategy — "stay on the lion OM and head directly toward M" — until time $t = \frac{\omega}{\log \omega} + \frac{\pi}{2}$. We can show that if the man keeps $|LM| > \rho^*$ during this interlude, then the lion will be able to force the distance $|LM|$ strictly below $\rho^*$ in a finite time. Hence the lion can achieve $\rho^*$ in a finite time when $\omega > 1$. This result is interesting because it is not true when $\omega = 1$. 
1. **Equal Speeds.**

First we obtain further results for the case $\omega = 1$ and suggest some open problems. Then we apply these results to the case $\omega > 1$.

We are interested in obtaining an upper bound on how long $M$ can keep the distance $|IM| \geq \epsilon > 0$ when $L$ starts at $0$.

**Lemma 1.1.** For any fixed $\epsilon > 0$

\[(1.1) \quad s(\epsilon) \leq \frac{1+\epsilon}{\sqrt{\epsilon}}\]

holds where

\[(1.2) \quad s(\epsilon) = \sup \{ t : |L(t)M(t)| \geq \epsilon, L(0) = 0 \text{ and } L \text{ uses the radial strategy} \}.\]

**Proof.** Select $\epsilon = \frac{1}{2^n}$ and $\delta = \frac{1}{2^n}$ for integers $n > m$. Let $N = \frac{1-\epsilon}{\delta}$.

Construct the following grid on the radial line passing through $M$:

$0, \delta, 2\delta, ..., N\delta = 1-\epsilon$.

Assume that $M$ tries to maintain $|IM| \geq \epsilon$ while $L$ uses the radial strategy. Using methods similar to those used in proving lemma 2.1 of [2], one can show that $L$ reaches $\delta$ by time $\epsilon \cdot \sin^{-1}(\frac{\delta}{\epsilon})$.

Similarly, starting from $\delta$, $L$ reaches $2\delta$ by time

$$(\epsilon+\delta)(\sin^{-1}(\frac{2\delta}{\epsilon+\delta}) - \sin^{-1}(\frac{\delta}{\epsilon+\delta})), \text{ etc.}$$

Hence $s(\epsilon)$ satisfies

\[(1.3) \quad s(\epsilon) \leq \sum_{n=0}^{N-1} (\epsilon+n\delta)(\sin^{-1}(\frac{(n+1)\delta}{\epsilon+n\delta}) - \sin^{-1}(\frac{n\delta}{\epsilon+n\delta})).\]
Letting $\delta \to 0$, the RHS of (1.3) approaches

$$\int_0^{1-\epsilon} \frac{\epsilon + x}{\sqrt{\epsilon^2 + 2\epsilon x}} \, dx \leq \frac{1+\epsilon}{\sqrt{\epsilon}}.$$

**Remark 1.1.** The following problem comes to mind. Suppose that the lion and man have equal speeds and that capture occurs when the distance $|IM|$ is less than some prescribed number $\rho_0$, the capture radius of the lion. How long can the man evade capture? What strategies are optimal for both players?

Denote by $F$ the intersection of the circumference $C$ and the halflines through $OL$. We are interested in upper bounds on how long $M$ can keep the distance $\frac{|LF|}{|MF|} \geq v > 1$ when $L$ starts at $O$.

**Lemma 1.2.** For any fixed $v > 1$

$$T(v) \leq \frac{2}{\log v}$$

holds where

$$T(v) = \sup \{ t \mid \frac{|L(t)F|}{|M(t)F|} \geq v, L(0) = 0 \text{ and } L \text{ uses the radial strategy} \}.$$

**Proof.** Let $L$ employ the radial strategy. Using methods similar to those used in lemma 1.1 one can show that

$$T(v) \leq \sum_{j=0}^{\infty} \left( 1 - \frac{1}{v^{j+1}} \right) \cos^{-1} \left( \frac{1 - \frac{1}{v^{j+1}}}{1 - \frac{1}{v^j}} \right)$$

$$\leq \sum_{j=0}^{\infty} \cos^{-1} \left( 1 - \frac{1}{v^j} \right).$$
But the RHS can be approximated by the integral

\[ \int_{0}^{\infty} \cos^{-1}(1 - \frac{1}{\sqrt{x}}) dx = \frac{K}{\log v} \]

where

(1.6) \[ K = \int_{0}^{\pi/2} \frac{\sin y}{1-\cos y} dy < 2. \]

**Definition 1.1.** Denote by \( M_\omega(t) \) that point on the line segment \( OM(t) \) which satisfies \( \omega |OM_\omega(t)| = |OM(t)| \).

**Remark 1.2.** There is a one to one correspondence between the game on the unit disc where \( L \) chases \( M \) moving at speed \( \omega \) and the game on the disc of radius \( \frac{1}{\omega} \) where \( L \) chases \( M_\omega \) moving at speed 1.

The next lemma applies our previous result to the case \( \omega > 1 \).

**Lemma 1.3.** Let \( \rho \) satisfy \( 0 \leq \rho \leq \frac{\omega-1}{\omega} \). If \( \omega > 1 \) and \( L \), starting at the center 0, employs the radial strategy until time \( \frac{2}{\log \omega} + \frac{\pi}{2} \), then either

(1.7) \[ |L(t)M(t)| \leq \rho \text{ for some } t \leq \frac{2}{\log \omega} \]

or

(1.8) \[ |OL(t)| > \frac{\rho}{\omega-1} \text{ for some } t \leq \frac{2}{\log \omega} + \frac{\pi}{2}. \]

**Proof.** Assume that \( |OM(t)| \leq \frac{\rho \omega}{\omega-1} \) for all \( t \leq \frac{2}{\log \omega} \). We are dealing with the situation where \( L \) chases \( M \) in a disc of radius \( \frac{\rho \omega}{\omega-1} \). By remark 1.2 this corresponds to \( L \) chasing \( M_\omega \) moving with speed 1
in a disc of radius \( \frac{\rho}{\omega-1} \). Let \( F \) denote the intersection of the line \( OM \) with the circle of radius \( \frac{\rho}{\omega-1} \) (see figure 1).

Now \( |L(t)M(t)| = |OM(t)| - |OL(t)| = \omega |OM_\omega(t)| - |OL(t)| = \omega \left( \frac{\rho}{\omega-1} - |M_\omega(t)| \right) - (\frac{\rho}{\omega-1} - |L(t)|) = \rho - \omega |M(t)| - |L(t)|. \) But lemma 1.3 implies that

\[
\frac{|L(t_0)|}{|M(t_0)|} \leq \omega \quad \text{where} \quad t_0 = \frac{2}{\log \omega}.
\]

Hence \( |L(t_0)M(t_0)| \leq \rho. \)

Clearly (1.7) holds a fortiori if

\[
|OM(t)| \leq \frac{2\omega}{\omega-1} \quad \text{for some} \quad \frac{2}{\log \omega} \leq t \leq \frac{2}{\log \omega} + \frac{\pi}{2}.
\]

Finally, it is not difficult to show that if \( |OM(t)| > \frac{2\omega}{\omega-1} \) for all \( \frac{2}{\log \omega} \leq t \leq \frac{2}{\log \omega} + \frac{\pi}{2} \), then \( |OL(t)| > \frac{\rho}{\omega-1} \) where \( t_1 = \frac{2}{\log \omega} + \frac{\pi}{2} \).

---

**Figure 1**
2. \( H(\rho) < \rho^* \) for \( \rho > \rho^* \).

Throughout the rest of this report we assume that \( \omega > 1 \). The following theorem follows from the results of section 1.

**Theorem 2.1.** If \( L_1 \), starting from the center 0, employs the radial strategy until time \( t_1 = \frac{2}{\log \omega} + \frac{\pi}{2} \), then either

\[
|L(t_1)M(t_1)| \leq \rho^*
\]

\[(2.1)\]

or

\[
|OL(t_1)| > \frac{\rho^*}{\omega-1}.
\]

\[(2.2)\]

**Proof.** Immediate from lemma 1.1.

Our goal is to show that if (2.2) holds, then \( L_1 \) can force the distance \( |LM| \) strictly below \( \rho^* \). In order to do so we must prove the following theorem.

**Theorem 2.2.** The function \( H \) (defined in section 3 of [3]) satisfies

\[
H(\rho) < \rho^* \text{ for } \rho > \rho^*.
\]

\[(2.3)\]

The remainder of this section is concerned with establishing Theorem 2.2.

We wish to study the situation when the game starts at the standard position \( (L_*, M_*) \) where \( |L_*M_*| = \rho_* \).

**Definition 2.1.** Suppose that the game starts at \( (L_*, M_*) \) and \( M \) employs the isometric strategy. For any \( \phi \in \Phi_2(L_*) \) let \( s_\phi = \inf\{t > 0 : \theta(t) = \beta\} \).

**Remark 2.1.** Clearly \( s_\phi < \infty \) for any radially increasing \( \phi \).
The following lemma follows easily from the results of sections 9 and 11 of [3].

**Lemma 2.1.** There exists a sequence of free strategies $\phi_j \in \Phi(L_*)$ such that

$$|OM_{\phi_j}^\delta(S_{\phi_j})| \to 1 \text{ as } j \to \infty$$

where both $M_{\phi_j}(t)$ and $L_{\phi_j}(t)$ are radially increasing and 0-convex for $0 \leq t \leq S_{\phi_j}$.

**Proof.** Easy.

**Definition 2.2.** We say that $\phi$ is $\ast$-optimal if $|OM_\phi(S_\phi)| = 1$.

**Lemma 2.2.** If there exists a $\ast$-optimal $\phi \in \Phi(L_*)$ then theorem 2.2 holds.

**Proof.** We have to show that if the game starts in the standard position $(L_0, M_0)$ where $|L_0 M_0| = \rho^* + \epsilon$ for some $\epsilon > 0$, then $L$ can force $|LM|$ strictly below $\rho^*$. 

Suppose that there exists a $\ast$-optimal strategy $\phi$. Then it is easy to show that there exists one for which both $L_{\phi_0}$ and $M_{\phi_0}$ are radially increasing and 0-convex on $0 \leq t \leq S_{\phi_0}$. But the 0-convexity of $M_{\phi_0}$ and the $\ast$-optimality of $\phi_0$ imply that the isometric strategy is the unique optimal strategy for $M$ against $\phi_0$, for any other strategy will cause the distance $|LM|$ to fall below $\rho^*$.

Specifically, if $M$ employs a strategy against $\phi_0$ which keeps $M$ colinear with $OL$ and radially increasing up to the time that $|LM| = \rho^* + \epsilon$ then $|LM|$ must eventually fall below $\rho^*$. 


Remark 2.2. Unfortunately it appears that there does not exist a *-optimal \( \phi_o \). Be Remark 5.2 of [3], the "obvious" way of determining \( \phi_o \) would be to find the \( \phi \) which minimizes the expression

\[
\frac{dy_2}{d\theta} = \rho_2 \cos \theta \begin{vmatrix} -\omega \cos(\theta + \psi) \\ -\sin(\theta + \phi) + \omega \sin(\theta + \psi)(1 - \frac{\rho \cos \psi}{1 - y_2}) \end{vmatrix}
\]

But if \( \theta = 0 \), \( \frac{dy_2}{d\theta} \to -\infty \) as \( \phi \to \frac{\pi}{2} \) while \( \frac{dy_2}{dt} = 0 = \frac{d\theta}{dt} \) for \( \phi = \frac{\pi}{2} \).

Fortunately we do not require that there exist a *-optimal \( \phi \).

The proof of theorem 2.2 follows.

Proof. All we have to show is that if the game starts at the standard position \( (L_0, M_0) \) with \( |LM_0| = \rho^* + \epsilon \) for some fixed \( \epsilon > 0 \), then \( L \) can force \( |LM| \) strictly below \( \rho^* \). Consider a game which starts at \( (L_*, M_*) \). By lemma 3.1 there exists a sequence \( \phi_n \in \Phi^* \) \( (n = 1, 2, \ldots) \) such that \( |OM_n(S_n)| \geq 1 - \frac{1}{n} \) and both \( L_n \) and \( M_n \) are radially increasing and 0-convex for \( 0 \leq t \leq S_n < \infty \).

Clearly we need only show that against some \( \phi_n \), there does not exist a strategy \( \psi_n \in \Psi^*(M_*) \) which keeps \( M \) collinear with \( OL \) and radially increasing until time \( t_n \) when \( |OL_n| = \rho^* + \epsilon \) and then continues to maintain \( |L_n M_n| \geq \rho^* \). Suppose such a \( \psi_n \) exists for each \( n \). Then the 0-convexity of \( M_n \) implies that the point \( M_n(t) \) lies in the region \( \mathcal{R}_n(t) \subset \mathcal{D} \) bounded by the arc of a circle of radius \( \rho^* \) and center \( L_0(t) \), the circumference of \( \mathcal{D} \), and the half-lines \( OL_n(t) \) and \( OM_n(t) \) for \( 0 \leq t \leq S_n \) (See Figure 2).
Now it is not difficult to show that there exists an integer $N$ and a constant $K > 0$ such that $|M_{\phi_n}(S_{\psi_n})M_{\psi_n}(S_{\phi_n})| \geq K$ uniformly in $n \geq N$. That $\|R_n(S_{\phi_n})\| = \sup\{|X-Y|: X, Y \in R_n(S_{\phi_n})\}$ $\to 0$ as $n \to \infty$ finishes the proof.

As a corollary we have the following result.

**Corollary 2.1.** The isometric strategy is the unique strategy which maintains $\rho^*$ at $(I, M)$. 

**Proof.** Immediate.

3. **The Lion Captures in a Finite Time.**

Now we are in a position to establish that the lion can achieve $\rho^*$ in a finite time. In this section we show that if the game starts in a position with $L$ lying on the line segment $OM$ and $|OL| > \frac{\rho^*}{a-1}$, then $L$ can force the distance $|LM|$ strictly below $\rho^*$ in a finite time.
Lemma 3.1. Given any standard position \((L_0, M_0)\) and \(\delta > 0\), there exists a strategy \(\phi_\delta \in \Phi(L_0)\) such that if \(L\) uses \(\phi_\delta\) and \(M\) uses \(\psi \in \Psi(M_0)\) then

\[
|L(t)M(t)| \leq H(|L_0M_0|) + \delta \quad \text{for some} \quad t \leq T_{\phi_\delta, \psi} < \infty.
\]

Proof. The result follows from definition 3.4 of [3].

An analogous result holds for starting positions \((L_0, M_0)\) where \(0, L_0\) and \(M\) are colinear.

Lemma 3.2. Suppose that the game starts in a position \((L_0, M_0)\) where \(L_0\) lies on the line segment \(0M_0\). For every \(\delta > 0\), there exists a \(\phi_\delta \in \Phi(L_0)\) such that if \(L\) uses \(\phi_\delta\) and \(M\) uses \(\psi \in \Psi(M_0)\) then

\[
|L(t)M(t)| < H(|M_0M_0|) + \delta + |L_0M_0|\]

holds for some \(t \leq T_{\phi_\delta, \psi} < \infty\).

Proof. Observe that the position \((\hat{L}_0, M_0)\) is a standard position where \(\hat{L}_0 = M_0\) (see definition 1.1). By lemma 3.1 there exists a strategy \(\phi_\delta \in \Phi(\hat{L}_0)\) such that for any fixed \(\psi \in \Psi(M_0)\)

\[
|\hat{L}(t)M(t)| \leq H(|M_0M_0|) + \delta
\]

for some \(t \leq T_{\phi_\delta, \psi} < \infty\).

Assume that there is an imaginary lion \(\hat{L}\) at \(\hat{L}_0\), which employs the strategy \(\phi_\delta\) against \(M\) and let \(L\) employ the radial strategy against that imaginary lion. Clearly
\begin{equation}
|\hat{L}(t)L(t)| \leq |\hat{L}_0 L_0| = |M_o L_o|.
\end{equation}

But by the triangle inequality

\[|L(t)M(t)| \leq |L(t)\hat{L}(t)| + |\hat{L}(t)M(t)|.\]

Hence the result follows from (3.3) and (3.4).

The next lemma contains our key result.

**Lemma 3.3.** Suppose that the game starts in a position \((L_o, M_o)\) where \(L_o\) lies on the line segment \(OM_o\) and satisfies \(|OL_o| = \frac{\rho_o}{\omega - 1} > \frac{\rho^*}{\omega - 1}\). Then \(L\) can force the distance \(|IM|\) to be strictly less than \(\rho^*\) in a finite time.

**Proof.** Theorem 2.2 implies that \(\lambda = \rho^* - H(\rho_o) > 0\). Now let \(L\) employ the radial strategy until time \(T_R\) when \(|IM_\omega| \leq \frac{\lambda}{2}\). Because of lemma 1.1 we know that \(T_R \leq \frac{\lambda}{\sqrt{\lambda}} < \infty\).

By lemma 3.2 there exists a \(\hat{\Phi}_{\lambda/2} \in \Phi(L(T_R))\) such that for each fixed \(\psi \in \Psi(M(T_R))\)

\begin{equation}
|L(t)M(t)| \leq H(|M_{\omega}(T_R)M(T_R)|) + \frac{\lambda}{2} + |L(T_R)M_{\omega}(T_R)|
\end{equation}

for some \(t\) satisfying \(t - T_R \leq \frac{\lambda}{\phi_{\lambda/2}, \frac{\lambda}{2}} < \infty\).

Clearly \(|M_{\omega}(T_R)M(T_R)| \geq \rho_o\). But by lemma 3.1 of [3], \(H\) is a non-increasing function on \(\rho \geq \rho^*\). Hence (3.5) becomes

\begin{equation}
|L(t)M(t)| \leq \rho^* - \frac{\lambda}{2}
\end{equation}
for some $t \leq T_R + T_{\varphi/5', \psi} < \infty$.

Theorem 3.1. The following strategy is optimal for the lion: First, go to center 0. Then employ the radial strategy until time $\frac{2}{\log \omega} + \frac{\pi}{2}$.

If $|IM|$ remains strictly greater than $\rho^*$ during that time interval, then the lion will be able to force $|IM|$ below $\rho^* - \varepsilon_{\psi}$ by time $T_{\psi}$

where $\varepsilon_{\psi} > 0$ and $T_{\psi} < \infty$ depend on the man's strategy $\psi$.

Proof. Immediate from Theorem 2.1 and Lemma 3.3.

4. Extensions.

In this section we modify the game by replacing our old payoff

$\lim \inf_{t \geq 0} |I(t)M(t)|$ with a new payoff $\inf_{t \geq 0} |I(t)M(t)|$. It is not difficult to show that this new game has a value $v(\cdot, \cdot)$ which is a continuous function from $\partial \times \partial'$ to $[0, \rho^*]$. Let us examine this value as a function of $|OM|$, the distance between M's initial position and the center 0, when L's initial position is fixed at 0.

As a convenience we will use the ambiguous notation $v = v(|OM|).

Proposition 4.1.

(4.1) $v(|OM|) = |OM|$ for $0 \leq |OM| \leq \rho^*$

(4.2) $v(|OM|) = \rho^*$ for $\rho^* \leq |OM| \leq \frac{2\omega \rho^*}{\omega - 1}$

(4.3) $v(|OM|) < \rho^*$ for $\frac{2\omega \rho^*}{\omega - 1} < |OM| \leq 1$. 

12
Proof. (4.1) is immediate. (4.2) and (4.3) follow from the fact that
if \(|OM| \geq \rho^{*}\) then \(M\) can maintain \(\rho^{*}\) iff \(M\) can make \(|OM| = \frac{\omega \rho^{*}}{\omega-1}\)
by the time \(L\) makes \(|OL| = \frac{\rho^{*}}{\omega-1}\).

Proposition 4.1 leads to the question: Does

\[
(4.4) \quad \frac{\omega \rho^{*}}{\omega-1} \geq \frac{1}{2} \quad \text{hold for all } \omega > 1.
\]

By examining the bounds \(\rho_{L}\) and \(\rho_{U}\) which appear in section 10 of [3], we see that (4.4) holds for moderate and large values of \(\omega\). However, we will have to examine matters more closely to settle the question.

Proposition 4.2. \(\frac{\omega}{\omega-1} \rho_{L}(\omega) \downarrow 0\) while \(\frac{\omega}{\omega-1} \rho_{U}(\omega) \downarrow \frac{1}{2}\) as \(\omega \downarrow 1\).

Proof. Easy

Conjecture 4.1. (4.4) does not hold for sufficiently small \(\omega\). Moreover, \(\frac{\omega \rho^{*}}{\omega-1}(\omega) \downarrow 0\) as \(\omega \downarrow 1\).

Remark 4.1. Unfortunately, I have not set up a computer program to calculate exact values of \(\rho^{*}(\omega)\). The output from such a program would make it easier to settle our question.

If Conjecture 4.1 is true, then \(L\) is penalized by starting at the circumference when \(\omega\) is small. This is quite consistent with results obtained for the case \(\omega = 1\) in [1].

Remark 4.2. As pointed out to me by L. Dubins, one of the essential differences between the problems of pursuit in the half plane [4] and pursuit in the circle is that in the former game it is always to the evaders advantage to move closer to the boundary.
REFERENCES


Given a lion (L) and a man (M) moving in a circular arena with constant speeds $l$ and $\omega > l$, respectively: How closely can L approach M and can he achieve a minimum distance? Besicovitch shows that L can get arbitrarily close to M but can never catch M when $\omega = l$. In a previous report we formulated the problem as a differential game, demonstrated that it has a value $\rho^*$, constructed upper and lower bounds on $\rho^*$ and determined an optimal evasion strategy for M.

Our main result is that the following strategy is optimal for L. Move to the center O. Then stay on the radial line OM and head toward M until time $\frac{2}{\log \omega}$. The resulting position is either one where $|LM| \leq \rho^*$ or one from which L can force $|LM|$ strictly below $\rho^*$ in a finite time. Hence for the case of unequal speeds the lion can achieve a minimum distance.
<table>
<thead>
<tr>
<th>KEY WORDS</th>
<th>LINK A</th>
<th>LINK B</th>
<th>LINK C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bounded Pursuit Game</td>
<td>ROLE</td>
<td>WT</td>
<td>ROLE</td>
</tr>
<tr>
<td>Differential Game</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pursuit</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Evasion</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Game</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lion and Man</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**INSTRUCTIONS**

1. **ORIGINATING ACTIVITY:** Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization (corporate author) issuing the report.

2a. **REPORT SECURITY CLASSIFICATION:** Enter the overall security classification of the report. Indicate whether the Restricted Data is included. Marking is to be in accordance with appropriate security regulations.

2b. **GROUP:** Autonomic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. **REPORT TITLE:** Enter the complete report title in all capital letters. Titles in all cases should be unclassified.

4. **DESCRIPTIVE NOTES:** If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final.

5. **AUTHOR(S):** Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initials.

6. **REPORT DATE:** Enter the date of the report as day, month, year; or month, year, if more than one date appears on the report, use date of publication.

7a. **TOTAL NUMBER OF PAGES:** The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7b. **NUMBER OF REFERENCES:** Enter the total number of references cited in the report.

8a. **CONTRACT OR GRANT NUMBER:** If appropriate, enter the applicable number of the contract or grant under which the report was written.

8b. **PROJECT NUMBER:** Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9a. **ORIGINATOR'S REPORT NUMBER(S):** Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9b. **OTHER REPORT NUMBER(S):** If the report has been assigned any other report numbers (either by the originator or by the sponsor), also enter this number(s).

10. **AVAILABILITY/LIMITATION NOTICES:** Enter any limitations on further dissemination of the report, other than those imposed by security classification, using standard statements such as:

1. "Qualified requesters may obtain copies of this report from DDC."

2. "Foreign announcement and dissemination of this report by DDC is not authorized."

3. "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through...

4. "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through...

5. "All distribution of this report is controlled. Qualified DDC users shall request through..."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. **SUPPLEMENTARY NOTES:** Use for additional explanatory notes.

12. **SPONSORING MILITARY ACTIVITY:** Enter the name of the sponsoring military department (e.g., the departmental project office or laboratory sponsoring the research and development). Include address.

13. **ABSTRACT:** Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as (T)4), (T)3), (T)2), (T)1), (C), or (U).

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. **KEY WORDS:** Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, roles, and weights is optional.