Speaker: Nicholas Cook, Stanford Mathematics

Title: Maximum of the characteristic polynomial for a random permutation matrix

Abstract:

Most of random matrix theory concerns the behavior of zeros of the characteristic polynomial (eigenvalues) in the limit as the size $N$ of the matrix tends to infinity. A big source of motivation is the universality phenomenon: that many point processes seemingly unrelated to matrices, such as zeros of the Riemann zeta function, behave like eigenvalues of random matrices. Recent work has moved beyond zeros to look at extreme values, which also show some universal behavior. In this talk I will consider the characteristic polynomial $\chi_N(z)$ for a uniform random $N \times N$ permutation matrix. Our main result is a law of large numbers for the maximum of $\log |\chi_N|$ over the unit circle. As in other works on extremes of log-correlated fields, our approach is to uncover a multi-scale structure in the distribution of $\chi_N$ and adapt a well-known second moment argument for the maximum of a branching random walk. Unlike the analogous problem for Haar unitary matrices, the distribution of $\chi_N(z)$ is sensitive to Diophantine properties of the argument of $z$. To deal with this we borrow tools from the Hardy–Littlewood circle method.

This is based on joint work with Ofer Zeitouni.