Speaker: Elina Robeva, MIT

Title: Statistical Estimation Under Algebraic Constraints: Total Positivity

Abstract:

Nonparametric density estimation is a challenging statistical problem: in general, the maximum likelihood estimate (MLE) does not even exist! While the MLE is known to exist under certain shape constraints such as log-concavity, statistical rates are still exponentially slow in the dimensionality of the data. In this talk we introduce total positivity as a constraint for density estimation, and we conjecture that it can alleviate the curse of dimensionality. Though they possess very special structure, totally positive random variables are quite common in real world data and exhibit appealing mathematical properties. Given i.i.d. samples from a log-concave and totally positive distribution, we prove that the MLE exists with probability one (for more than three samples). We characterize the domain of the MLE, and give algorithms to compute it. If the observations are two-dimensional or binary, we show that the logarithm of the MLE is a piecewise linear function and can be computed via a certain convex program. Finally, we prove statistical guarantees for the convergence of the MLE in two dimensions. This highlights one vein of my broader work on statistical estimation under algebraic constraints.