The Adaptive Lasso and Its Oracle Properties

The lasso (or $\ell_1$ penalization) is famous for its ability to automatically produce a sparse subset model. It has been shown that the $\ell_1$ penalization approach can discover the "right" sparse representation of the model under certain conditions (e.g., orthogonal design). The LARS algorithm (Efron, Hastie, Johnstone and Tibshirani 2004, Annals of Stat.) further facilitates the applications of the lasso in practice. The $\ell_1$ shrinkage method now seems to become the default choice for building a sparse model.

In this talk, we first show a necessary condition for the lasso variable selection to be consistent. We present some examples in which the lasso is inconsistent for variable selection. In the second part of the talk, we propose a new version of the lasso, called the adaptive lasso, where adaptive weights are used for penalizing different coefficients in the $\ell_1$ penalty. We show that the adaptive lasso is consistent in variable selection, and in addition, it possesses the oracle properties using the language in Fan and Li (2001, JASA). A new oracle inequality is derived to show that the adaptive lasso shrinkage is near minimax optimal in the sense of Donoho and Johnstone (1994, Biometrika). As a byproduct of our analysis, Breiman's nonnegative garotte is shown to be consistent for variable selection.

If time permits, we will also discuss the extension of the adaptive lasso in generalized linear models, survival models and the support vector machines.