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AN OPTIMAL SEQUENTIAL ACCELERATED LIFE TEST WITH EXPONENTIAL DEPENDENCE ON STRESS

By

GIDEON SCHWARZ

TECHNICAL REPORT NO. 67

December 12, 1960

PREPARED UNDER CONTRACT Nonr-225(52)
(NR-342-022)
FOR
OFFICE OF NAVAL RESEARCH

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Summary:

In [1] Bessler, Chernoff and Marshall present a method for obtaining optimal sequential designs for experiments to test whether a device subjected to standard stress has an expected lifetime exceeding a specified value. It is assumed there that the lifetime is exponentially distributed, with expectation a function of the stress and an unknown parameter. The case in which that function is the inverse of a quadratic is solved in [1] explicitly.

In the present paper, the same method is applied to the case where the dependence of the expected lifetime on the stress is exponential.

Formulation of the game:

A random variable T, the life-time, is exponentially distributed on the positive half-line, and its unknown mean μ is related to the stress x and to two unknown parameters φ_1 and φ_2 through the formula $\mu = e^{-\varphi_1 x - \varphi_2}$. We wish to test a one-sided hypothesis about the mean lifetime at "normal stress". The stress scale can be chosen so that normal stress = zero, and the time scale so that we are testing $\mu \geq 1$ against $\mu \leq 1$. With those choices, the hypotheses are equivalent to $\varphi_2 \leq 0$ and $\varphi_2 \geq 0$ respectively. We assume that φ_2 can take on any value, while φ_1 is restricted to non-negative values. The available stress-levels x are assumed to be all numbers up to a positive given maximal stress K.

According to [1], the choice of the stress level for the next experiment is determined by an optimal strategy for the first (i.e., maximizing) player in a game (X, Y, F), in which the available strategies for the first player are the stress levels, and for the second player, the parameter points in the hypothesis alternative to the one including (θ_1, θ_2), the maximum likelihood estimate of (φ_1, φ_2) from the observations taken so far. The payoff is given by

$$F(x, \varphi_1, \varphi_2) = \frac{I(x, \theta_1, \theta_2, \varphi_1, \varphi_2)}{\mu(x, \theta_1, \theta_2)} .$$

Here $I(x, \theta_1, \theta_2, \varphi_1, \varphi_2)$ is the Kullback-Leibler information number (cf. [2]) between the distribution of T with $\mu(x, \theta_1, \theta_2)$ and with $\mu(x, \varphi_1, \varphi_2)$.

Denoting $\varphi = \varphi_1 x + \varphi_2$ and $\theta = \theta_1 x + \theta_2$ we have $\theta - T e^{\theta}$ and $\varphi - T e^{\varphi}$ for the logarithms of the two densities. We therefore have $I(x, \theta_1 \theta_2, \varphi_1 \varphi_2) = \theta - \varphi + e^{-\theta}(e^{\varphi} - e^{\theta})$ and, finally

$$F(x, \varphi_1, \varphi_2) = e^{\theta}[e^{\varphi-\theta} - \varphi + \theta - 1] .$$

Solution of the game:

We start by showing that the set of nature's strategies can be restricted to the set of parameter points on the common boundary of the hypotheses. Denoting the expression of $F(x, \varphi_1, \varphi_2)$ in terms of φ and θ by $F(\varphi, \theta)$, we have $\frac{\partial^2 F}{\partial \varphi^2} = e^{\varphi} > 0$. Thus F is convex in φ, and as φ is linear in (φ_1, φ_2), F is convex in (φ_1, φ_2). From its definition as a ratio of two non-negative quantities, F is non-negative. At the point $(\varphi_1, \varphi_2) = (\theta_1, \theta_2)$ F is zero, and therefore it can only increase along any ray starting from (θ_1, θ_2). Thus, for any strategy (φ_1, φ_2) the point at which the interval from (φ_1, φ_2) to (θ_1, θ_2) intersects the boundary $\varphi_2 = 0$, will have smaller or equal payoff than (φ_1, φ_2) for any x. The points on the boundary can be written as $(y, 0)$.

The game is thus reduced to a game in the plane, with $X = \{x | -\infty < x \leq K\}$ $Y = \{y | 0 < y < \infty\}$ and payoff function, which we again denote by F,

$$F(x, y) = e^{\theta}[e^{\varphi-\theta} - \varphi + \theta - 1]$$

were now $\theta = \theta_1 \, x + \theta_2$ and $\varphi = x\,y$. We shall also use z, to denote $\varphi - \theta = (y - \theta_1)\,x - \theta_2$. We then have $F = e^{\theta}(e^{z} - z - 1)$, and $\frac{\partial z}{\partial x} = y - \theta_1$, and $\frac{\partial z}{\partial y} = x$.

In order to solve the game, we determine the points were for constant y, F attains a maximum with respect to x.

Differentiating F with respect to x we have

$$\frac{\partial F}{\partial x} = [(e^{z} - 1)\,y - \theta_1\, z]\,e^{\theta} \quad .$$

Setting $\frac{\partial F}{\partial x}$ equal to zero, we find $(e^{z} - 1)\,y = \theta_1\, z$, hence the derivative is zero when $z = 0$ and also when

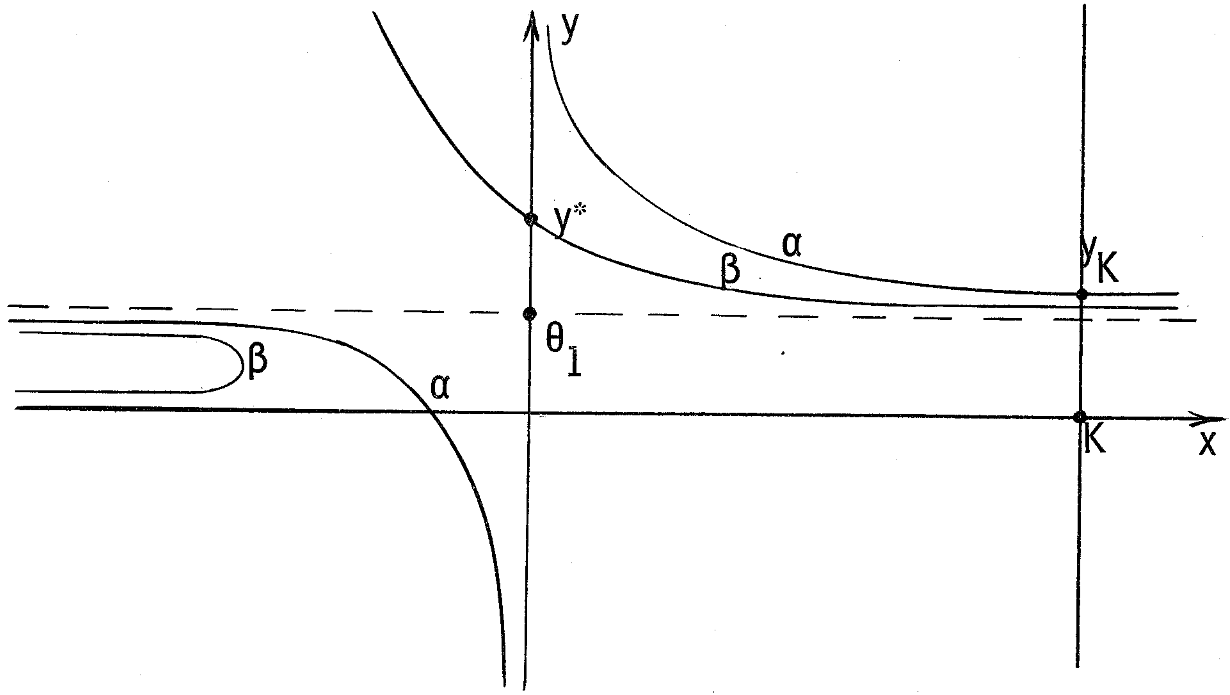
$$y = \frac{\theta_1\, z}{e^{z} - 1} \quad .$$

The first curve, $z = 0$, is a hyperbola $(x - k_o)(y - \theta_1) = \theta_2$. On this hyperbola $F(x, y) = 0$, and as F is non-negative, it has a minimum at every point of the hyperbola. The second curve, given by $y = \theta_1 z/(e^z - 1)$ intersects every line $y =$ constant > 0 exactly once, with the exception of the the line $y = \theta_1$, which it never intersects. This follows from the fact that the function $z/(e^z - 1)$ is defined, positive and decreasing for all z. Its inverse function $g(t)$ is defined for all $t > 0$, and the equation $z = g(y/\theta_1)$ assignes a z to every positive y. As we have $z = (y - \theta_1)(x - k_o) - (\theta_1 k_o + \theta_2)$ there is a unique x correspon-ding to given y and z, when $y \neq \theta_1$; that x is the abscissa of the unique intersection of $y =$ constant $\neq \theta_1$ with $y = \theta_1 z/(e^z - 1)$. On the other hand, if $y = \theta_1$, we have $z = -\theta_2 \neq 0$, while $y = \theta_1 z/(e^z - 1)$ can equal θ_1 only for $z = 0$.

As every line $y =$ constant > 0 intersects both the hyperbola and the curve $y = \theta_1 z/(e^z - 1)$, except the line $y = \theta_1$, and as the curve and the hyperbola never intersect, the payoff function has a unique local maximum on every line $y = a \neq \theta_1$, and that maximum is attained at $x(a)$, the solution of

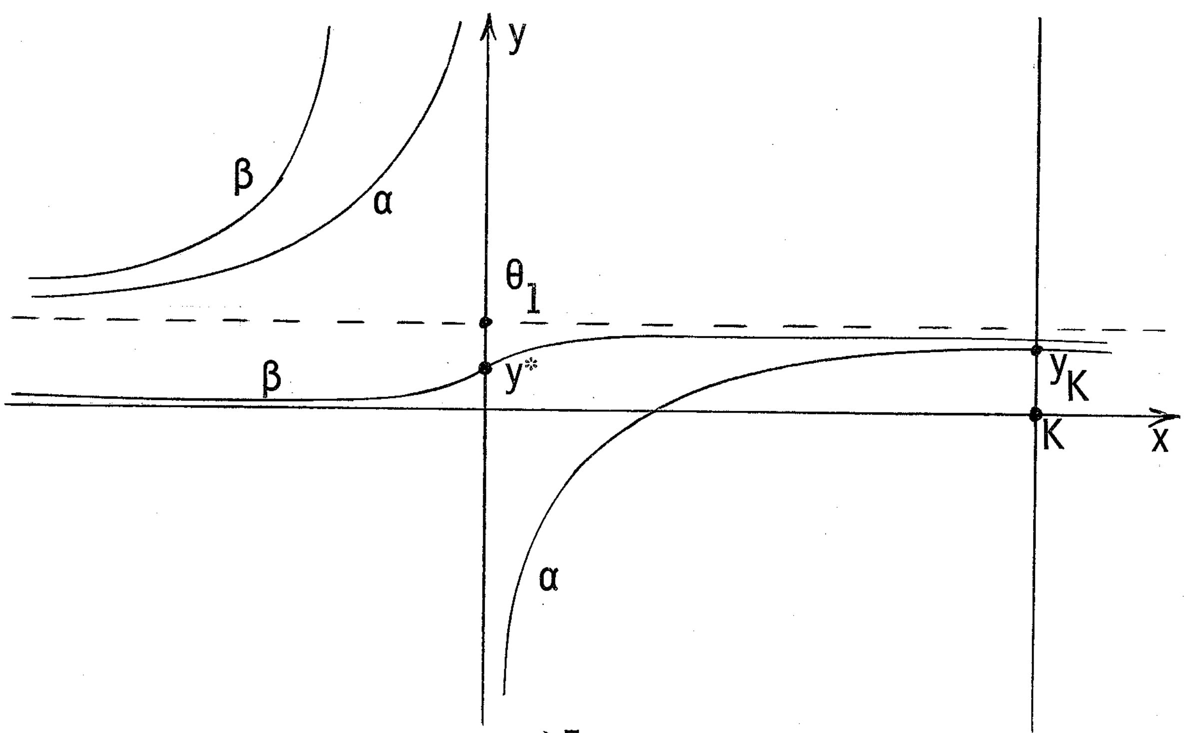
$$a = \theta_1 \, z(x, a)/(e^{z(x, a)} - 1) .$$

As for x decreasing indefinitly $F(x, y)$ tends to zero, the behaviour of $F(x, y)$ along $y = a \neq \theta_1$ is as follows: it increases when x increases from $-\infty$ to $x(a)$, then it decreases till the hyperbola is



The locus of $\frac{\partial F}{\partial x} = 0$ when $\theta_2 > 0$

α - the hyperbola
β - the curve $y = \theta_1 \, z/(e^z - 1)$



The locus of $\frac{\partial F}{\partial x} = 0$ when $\theta_2 < 0$

α - the hyperbola
β - the curve $y = \theta_1 \, z/(e^z - 1)$

crossed, after which it increases again. As x is limited to values less than or equal to K, the absolute maximum of $F(x, a)$ is attained either at $(x(a), a)$ or at (K, a).

In general, a line $x =$ constant may cross the curve $y = \theta_1 z/(e^z - 1)$ more than once, or not at all; however, the line $x = 0$ crosses the curve exactly once. For $x = 0$ we have $z = -\theta_2$, which is independent of y, and the unique intersection with the curve is at

$$(*) \qquad y^* = \frac{-\theta_1 \theta_2}{e^{-\theta_2} - 1} .$$

Now consider the two lines $x = 0$ and $y = y^*$. On the first one z, and therefore $F(x, y)$, are constant. On the second one, the maximum of $F(x, y)$ is attained at $(0, y^*)$ or at (K, y^*). Assuming $F(K, y^*) < F(0, y^*)$, the maximum is attained at $(0, y^*)$, and the point $(0, y^*)$ is a saddle-point solution to the game. It is then a unique solution, because minima with respect to y are obtained for $x \neq 0$ only on the hyperbola, and on the hyperbola $F \equiv 0$, while $F(0, y^*) \neq 0$, and F must be the same at all saddle-points.

Summarizing, we see that y^* as defined by $(*)$ is the unique optimal strategy for nature when $F(0, y^*) > F(K, y^*)$, and in that case $x = 0$, that is, using the normal stress is the unique optimal strategy for the statistician.

Let us now consider the alternative case, that is, $F(0, y^*) \leq F(K, y^*)$. The geometrical configuration depends on whether θ_2 is positive or negative, but we shall treat both cases simulatneously by adding the words appropriate to the case of negative θ_2 in parentheses whenever necessary. First, from the definition of y^*, y^* is positive (negative), and as the hyperbola is confined to the first and third (second and fourth) quadrants, the line $y = y^*$ intersects the hyperbola in the first (fourth) quadrant, that is, for some $x_o > 0$. The term "quadrant" is used here in terms of the coordinates x and $y - \theta_1$ (cf. figure). As $F(x, y^*)$ has no stationary points except 0 and x_o , we have $0 < F(x, y^*) < F(0, y^*)$ for $0 < x < x_o$, and the assumed inequality $F(K, y^*) \geq F(0, y^*)$ is possible only if $K \geq x_o$. The point (K, y^*) is therefore on or above (on or below) the hyperbola. Equivalently, if we denote by y_K the ordinate of the intersection of the hyperbola with the line $x = K$, y_K will be greater or equal (smaller or equal to y^*). We shall now prove that some number between y_K and y^* is the unique optimal strategy for nature. As $F(x, y)$ is convex in y , such a strategy exists, and equals that value of y for which $\max_x F(x, y)$ attains its minimum. We start therefore by studying the behaviour of $F(x, y)$ on the line $x = K$ and on the curve $y = \theta_1 z/(e^z - 1)$, the line and the curve being the only locations where maxima with respect to x can be attained.

First, on the line $x = K$, $F(x, y) \equiv F(K, y)$ is convex and therefore increasing for $y > y_K$ and decreasing for $y < y_K$. In particular, in the interval with end-points y_K and y^* , $F(K, y)$ is increasing (decreasing).

Second, on the curve $y = \theta_1 z/(e^z - 1)$ the partial derivative $\frac{\partial F}{\partial x}$ vanishes, and $\frac{dF}{dy} = \frac{\partial F}{\partial y}$; hence, moving along the curve by increasing y , F is increasing above the hyperbola and decreasing below it.

Combining those two facts, we see that for values of y greater (smaller) than y^*, both $F(K, y)$ and F along the curve are increasing (decreasing), and it is $\max_x F(x, y)$; for values of y smaller (greater) than y_K , both are decreasing (increasing), and, again, so is $\max_x F(x, y)$. Accordingly, nature's best strategy must be found between y_K and y^* .

To obtain an equation for nature's best strategy $\hat{y}$ we observe that as along the line $x = K$, the function F increases from zero to $F(K, y^*) > F(0, y^*)$ as y goes from y_K to y^* (decreases from $F(K, y^*) > F(0, y^*)$ to zero as y goes from y^* to y_K), while along the curve, for the same values of y , F increases to $F(0, y^*)$ (decreases from $F(0, z^*)$) , the two must meet at some y in between, and for that y , $\max_x F(x, z)$ attains its minimum. Equating $F(x, y)$ on the line $x = K$ and on the curve $y = \theta_1 z/(e^z - 1)$, we obtain the following procedure for calculating $\hat{y}$: denote again by $g(t)$ the inverse function of $z/(e^z - 1)$, the x-coordinate of a point on $y = \theta_1 z/(e^z - 1)$ is given by

$$\frac{g(y/\theta_1) + \theta_2}{y - \theta_1}$$

and the number $\hat{y}$ is the solution of the equation
$F(K, y) = F(\frac{g(y/\theta_1) + \theta_2}{y - \theta_1}, y)$ that lies between y_K and y^* .

Once $\hat{y}$ has been found, the optimal strategy for the statistician is easily obtained.

It will consist, according to theorem 12.5 of [3], of randomizing between the two values of x for which $F(x, \hat{y})$ is maximal, that is, the values K and

$$x = \frac{g(\hat{y}/\theta_1) + \theta_2}{\hat{y} - \theta_1} ,$$ with weights whose ratio equals

$$- \frac{\partial F(\hat{x}, \hat{y})}{\partial y} \Bigg/ \frac{\partial F(K, \hat{y})}{\partial y} .$$

As (x, y) and (K, y) lie on different sides of the hyperbola, this ratio is positive, as required by theorem 12.5 of [3]

References

[1] Bessler, S., Chernoff, H., and Marshall, A.W. "An optimal sequential accelerated life test", Contract Nonr 225(52) No. 65, Stanford University 1960

[2] Kullback S., Information theory and statistics, John Wiley, N.Y., 1959.

[3] McKinsey, J.C.C, Introduction to the theory of games, Mcgraw-Hill, N.Y., 1952.

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