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STORAGE POLICY FOR INCOMPLETE RECORD-KEEPING

By

HERMAN CHERNOFF and GIDEON SCHWARZ

TECHNICAL REPORT NO. 71

July 2, 1961

**PREPARED UNDER CONTRACT Nonr-225(52)
(NR-342-022)**

**FOR
OFFICE OF NAVAL RESEARCH**



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1. Introduction

Let us consider a depot which stores and supplies a certain item. When a demand or request for this item is submitted, it is met by the depot. The transaction is recorded and the balance (the amount recorded to be in storage) is diminished by the amount supplied. However, it is possible that, by oversight, the transaction fails to be recorded.

When the balance falls below a certain number called the reorder point the item is inventoried and an order is placed to restock the item to a level considered to be a full supply. Because of possible failures to record transactions it may be that the stock (amount of the item actually on hand) is considerably less than the balance. Thus there may fail to be sufficient stock to supply the demand even though the recorded balance is greater than the reorder point. In that case the stock is reordered but the inability to supply the demand leads to a loss whose average value c may be substantial.

If the reorder point is low, the unfilled demand may prove costly. On the other hand, if it is high, there will be unnecessarily frequent inventories and reorderings which also lead to considerable cost. It is desirable to evaluate the expected cost per transaction and to see how it depends on the choice of the reorder point. This evaluation can then be

applied to find an optimal reorder point. Approximate evaluations are presented graphically in the figures at the end of this paper.

In section 2, the problem of finding an optimal reorder point is presented formally and the graphs which indicate approximations to the solution are described.

The remaining sections are devoted to presenting the formulae used to obtain the graphs and their derivations. Two distinct cases are treated in the derivations. The case where the demand is constant is treated in section 3. For the other case the demand is a random variable with known mean and variance. The derivations cover sections 4, 5 and 6. The results are summarized in section 6. The numbered equations of sections 5 and 6 may be used to evaluate the expected cost per transaction.

2. Formal Description of the Problem and Results

Let s represent a full supply of the item and let b^* be the reorder point. Suppose that successive demands for the item may be regarded as independent observations on a random variable X with mean μ and variance σ^2 .

Suppose that each transaction may fail to be recorded with probability p^* and such failures are independent events. Finally let the cost of reordering be one and the average cost of failing to supply a demand be c (plus one for reordering).

Using these assumptions, we may evaluate an approximation to L , the average cost per transaction. In the graphs at the end of this paper we present L as a function of b^* when $\mu = 1$ for various values of σ , c , p^* and s . In each case the minimizing value of b^* and L_{\min} the corresponding value of L are indicated.

Note that L is unaffected if stocks are measured in units obtained by selecting $\mu = 1$. If one prefers to use another unit, this statement implies that L is unaltered when μ , σ , s and b^* are replaced by 1 , σ/μ , s/μ and b^*/μ respectively.

We shall find the following terms useful. Let N represent the average number of transactions before reordering. Let P be the probability that starting with a full supply, there will be an unfilled demand before reordering. Let $p = 1 - p^*$ be the probability of recording a transaction and $b = s - b^*$. Finally we shall use ϕ and Φ to represent the density and cumulative distribution functions for the normal distribution with mean 0 and standard deviation one.

3. The case of fixed-size demands

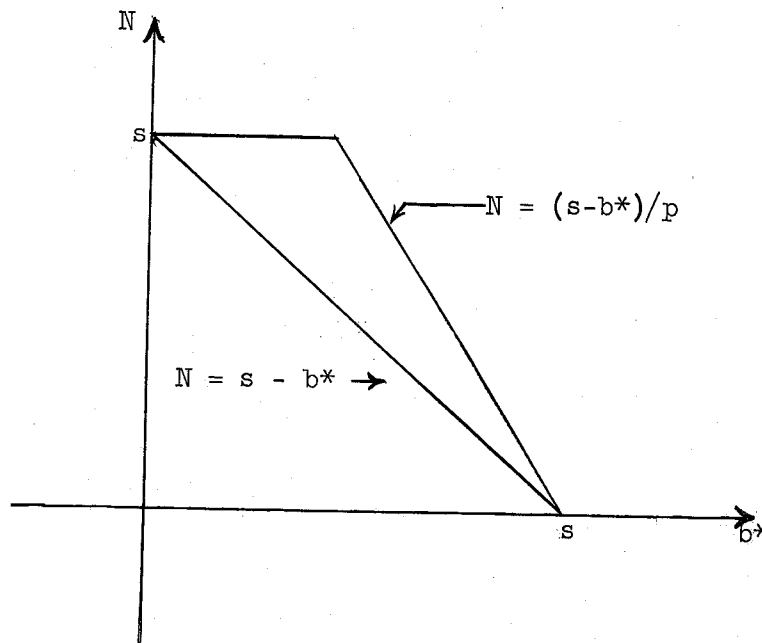
Let us assume that each demand is for just one unit, and that the probability of a transaction being recorded is $p = 1 - p^*$, independently of the recording of other transactions. The storage center will fail to satisfy the $s + 1$ st demand if and only if fewer than $b = s - b^*$ demands out of the first s demands were recorded. Equivalently, this occurs whenever no more than $b - 1$ transactions were recorded before the b^* th omission. As p^* is the probability of an omission, the number of recordings preceding the b^* th omission has a Laplace (Negative Binomial [1]) distribution with parameters p^* and b^* . Denoting the sum of the n first terms in this distribution by $Y(n, b^*)$ we obtain

$$P = Y(b-1, b^*)$$

for the probability of being caught with our supply empty.

So far no approximations have been made. However, when we come to evaluate the second aspect of employing a certain reorder point, namely the average frequency of reordering to which it leads, the exact result involves means of truncated Laplace distribution, and for the sake of simplicity we apply the following approximations:

If reordering were always due to the stock records reaching the reorder point, the average number N of transactions until reordering would be $b/p = (s-b^*)/(1-p^*)$. The true average number is less than that, but never less than b . Also N can never exceed s . Thus we have a graph of N as a function of b^* enclosed in the following triangle.



As we shall see that the best choice of b^* is either zero, or else it is a value for which the probability of running out is quite small. In that case we may approximate N by $(s-b^*)/p$. Hence using this approximation for all b^* except at 0, where we shall use the true value $N = s$, yields results which are rather accurate for the optimal reorder points. Consequently this approximation furnishes an accurate estimate of the optimal reorder point.

The approximate overall average loss per transaction is now given for $b^* > 0$ by

$$L = \frac{1 + cP}{N} = \frac{1 + cY(b-1, b^*)}{s - b^*} (1-p^*) .$$

When $b^* = 0$ we can avoid running out only by recording every single transaction, and we have

$$L = \frac{1 + c[1 - (1-p^*)^s]}{s} .$$

The graph of $L(b^*)$ can be easily drawn with the aid of a table of the cumulative binomial $B(k, n)$ if we make use of the relation $Y(k, n) + B(n-1, n+k) = 1$ (see [2]). It is especially convenient due to the fact we need the values

$$Y(s-b^*-1, b^*) = 1 - B(b^*-1, s-1)$$

which, for fixed s , are all found in the same table (see for instance [3]).

The graphs pertaining to the case of fixed-size demand were obtained with a desk computer and can be identified by the heading " $\sigma = 0$ ".

4. Preliminaries for the general case

Let X_1, \dots, X_n, \dots , and X_1^*, \dots be independent identically distributed random variables with mean μ and variance σ^2 . They will represent the demands where each recorded transaction is labelled as an X , and each omitted one is labelled as X^* .

Let Z_r be the number of transactions that are recorded before the r^{th} one that is not, and Z_r^* be the number that are not recorded before the r^{th} one that is. Clearly Z_r and Z_r^* are dependent random variables, each of which has a negative binomial distribution.

Let R_a and R_a^* be defined by

$$X_1 + X_2 + \dots + X_{R_a} \geq a > X_1 + X_2 + \dots + X_{R_a-1}$$

and

$$X_1^* + X_2^* + \dots + X_{R_a^*}^* \geq a > X_1^* + X_2^* + \dots + X_{R_a^*-1}^* .$$

For convenience we shall assume that an unfilled demand arises only if the total of forgotten transactions is at least b^* . This assumption involves the neglect of an event whose probability approaches zero as b and b^* approach ∞ .

Then, the event of an unfilled demand is equivalent to

$$V = Z_{R_b^*} - R_b < 0$$

and to

$$V^* = Z_{R_b}^* - R_{b^*}^* \geq 0 .$$

If there is an unfilled demand, the total number of transactions leading to this demand is

$$W = R_{b^*}^* + Z_{R_{b^*}^*} .$$

On the other hand, if the reorder point is reached first, the total number of transactions before reordering is

$$W^* = R_b + Z_{R_b}^* .$$

The average cost associated with reordering is $1 + cP\{V < 0\}$. The average number of transactions leading to reordering is

$$N = E(W|V < 0) P(V < 0) + E(W^*|V^* < 0) P(V^* < 0)$$

and the average cost per transaction is

$$L = [1 + cP(V < 0)]/N .$$

In the following sections we shall derive asymptotic expressions for $P(V < 0)$, and N .

5. Asymptotic Approximations

(a) Negative Binomial

If Z_r is the number of transactions that are recorded before the r^{th} that isn't,

$$P\{Z_r \leq z\} = P\{Y \leq z\}$$

where Y is the number of successes of an event with probability p out of $z+r$ independent trials.

$$\therefore P\{Z_r \leq z\} \doteq \Phi \left(\frac{z - (r+z)p + \frac{1}{2}}{\sqrt{(r+z)p p^*}} \right)$$

where Φ is the cumulative normal distribution with mean 0 and variance 1. Furthermore $Z_r/r \rightarrow p/p^*$ in probability. It follows that Z_r is approximately normally distributed with mean $\frac{rp}{p^*} - \frac{1}{2p^*}$ and variance $\sqrt{r[1+(p/p^*)] \frac{p}{p^*}}$ or more precisely

$$Z_r = \left(\frac{rp}{p^*} - \frac{1}{2p^*} \right) + \epsilon_1 \sqrt{r \left(1 + \frac{p}{p^*} \right) \frac{p}{p^*}}$$

where ϵ_1 is asymptotically normal with mean 0 and variance 1 for large r .

^{1/} The $-\frac{1}{2p^*}$ term is relatively small for large r but was included to improve the approximation.

(b) The "waiting time" R_a

Applying the central limit theorem, we have

$$\begin{aligned} P\{R_a \leq r\} &= P\{X_1 + X_2 + \cdots + X_r \geq a\} \\ &= 1 - \Phi\left(\frac{a - r\mu}{\sqrt{r}\sigma}\right) = \Phi\left(\frac{r - \frac{a}{\mu}}{\sqrt{r}\sigma/\mu}\right). \end{aligned}$$

Since $\frac{R_a}{a}$ converges in probability $1/\mu$ as $a \rightarrow \infty$, it follows that

$$R_a = a/\mu + \sqrt{\frac{a\sigma^2}{\mu^3}} \epsilon_2$$

where ϵ_2 is asymptotically normal with mean 0 and variance 1 as $a \rightarrow \infty$.

(c) $Z_{R_{b^*}}^*$, V , W

Combining (a) and (b), we have

$$\begin{aligned} Z_{R_{b^*}}^* &= \left(R_{b^*}^* \frac{p}{p^*} - \frac{1}{2p^*}\right) + \epsilon_1 \sqrt{R_{b^*}^* \left(1 + \frac{p}{p^*}\right) \frac{p}{p^*}} \\ &= \frac{b^*}{\mu} \frac{p}{p^*} - \frac{1}{2p^*} + \frac{p}{p^*} \frac{b^*\sigma^2}{\mu^3} \epsilon_2^* + \epsilon_1 \sqrt{\frac{b^*}{\mu} \left(1 + \frac{p}{p^*}\right) \frac{p}{p^*}}. \end{aligned}$$

Now

$$V = Z_{R_{b^*}}^* - R_b = v_0 + v_1 \epsilon_1 + v_2 \epsilon_2 + v_3 \epsilon_2^*$$

where

$$\begin{aligned}v_0 &= \frac{b^*}{\mu} \frac{p}{p^*} - \frac{1}{2p^*} - \frac{b}{\mu} \\v_1 &= \sqrt{\frac{b^*}{\mu} \left(1 + \frac{p}{p^*}\right) \frac{p}{p^*}} \\v_2 &= -\sqrt{\frac{b\sigma^2}{\mu^3}} \\v_3 &= \frac{p}{p^*} \sqrt{\frac{b^*\sigma^2}{\mu^3}}\end{aligned}\tag{5.1}$$

Finally

$$W = R_{b^*}^* + Z_{R_{b^*}^*} = w_0 + w_1 \epsilon_1 + w_2 \epsilon_2 + w_3 \epsilon_2^*$$

where

$$\begin{aligned}w_0 &= \frac{b^*}{\mu} \left(1 + \frac{p}{p^*}\right) - \frac{1}{2p^*} \\w_1 &= \sqrt{-\frac{b^*}{\mu} \left(1 + \frac{p}{p^*}\right) \frac{p}{p^*}} \\w_2 &= 0 \\w_3 &= \left(1 + \frac{p}{p^*}\right) \sqrt{\frac{b^*\sigma^2}{\mu^3}}\end{aligned}\tag{5.2}$$

It follows that (V,W) has approximately a bivariate normal distribution with means and covariances given by

$$\mu_V = v_0 ; \quad \mu_W = w_0 ,$$

$$\sigma_V^2 = v_1^2 + v_2^2 + v_3^2, \quad \sigma_W^2 = w_1^2 + w_3^2$$

$$\sigma_{VW} = w_1 v_1 + w_3 v_3 .$$

Assuming that V and W do have a joint normal distribution, we are interested in $P(V < 0)$ and $E(W|V < 0)$.

First

$$P(V < 0) = P \left(\frac{V - \mu_V}{\sigma_V} < \frac{-\mu_V}{\sigma_V} \right) = \Phi \left(\frac{-\mu_V}{\sigma_V} \right) .$$

Then

$$E(W|V) = \mu_W + \frac{\sigma_{WV}}{\sigma_V^2} (V - \mu_V) = \mu_W + \frac{\sigma_{WV}}{\sigma_V} \left(\frac{V - \mu_V}{\sigma_V} \right)$$

$$E(W|V < 0)P(V < 0) = \mu_W P(V < 0) + \frac{\sigma_{WV}}{\sigma_V} \int_{-\infty}^{\frac{-\mu_V}{\sigma_V}} x \phi(x) dx$$

$$= \mu_W \Phi \left(\frac{-\mu_V}{\sigma_V} \right) - \phi \left(\frac{-\mu_V}{\sigma_V} \right) \frac{\sigma_{WV}}{\sigma_V}$$

6. Final Results

Combining the equations of the preceding sections, we have the following approximations:

$$(6.1) \quad P(V < 0) = \Phi(k_1)$$

where

$$(6.2) \quad k_1 = -v_0 / \sqrt{v_1^2 + v_2^2 + v_3^2} .$$

$$(6.3) \quad E(W|V < 0)P(V < 0) = w_0 \Phi(k_1) - k_2 \phi(k_1) = \psi(\mu, \sigma^2, b, b^*, p, p^*)$$

where

$$(6.4) \quad k_2 = (w_1 v_1 + w_3 v_3) / \sqrt{v_1^2 + v_2^2 + v_3^2} .$$

By symmetry, we have

$$E(W^*|V^* < 0)P(V^* < 0) = \psi(\mu, \sigma^2, b^*, b, p^*, p) .$$

Hence the expected number of transactions until reordering is

$$(6.5) \quad N = \psi(\mu, \sigma^2, b, b^*, p, p^*) + \psi(\mu, \sigma^2, b^*, b, p^*, p)$$

and the average cost per transaction is

$$(6.6) \quad L = [1 + c\Phi(k_1)]/N .$$

The numbered equations of sections 5 and 6 may be used to compute L, N, and P.

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- [1] William Feller, An Introduction to Probability Theory and its Applications, Wiley and Sons, N.Y. (1957), p. 155 .
- [2] G. P. Patil, "On Evaluation of Negative Binomial Distribution Function", Ann. Math. Stat. 31 (2) (1960) p. 527.
- [3] Tables of the Cumulative Binomial Probability Distribution, Harvard University Press, Cambridge (Mass.) (1955).

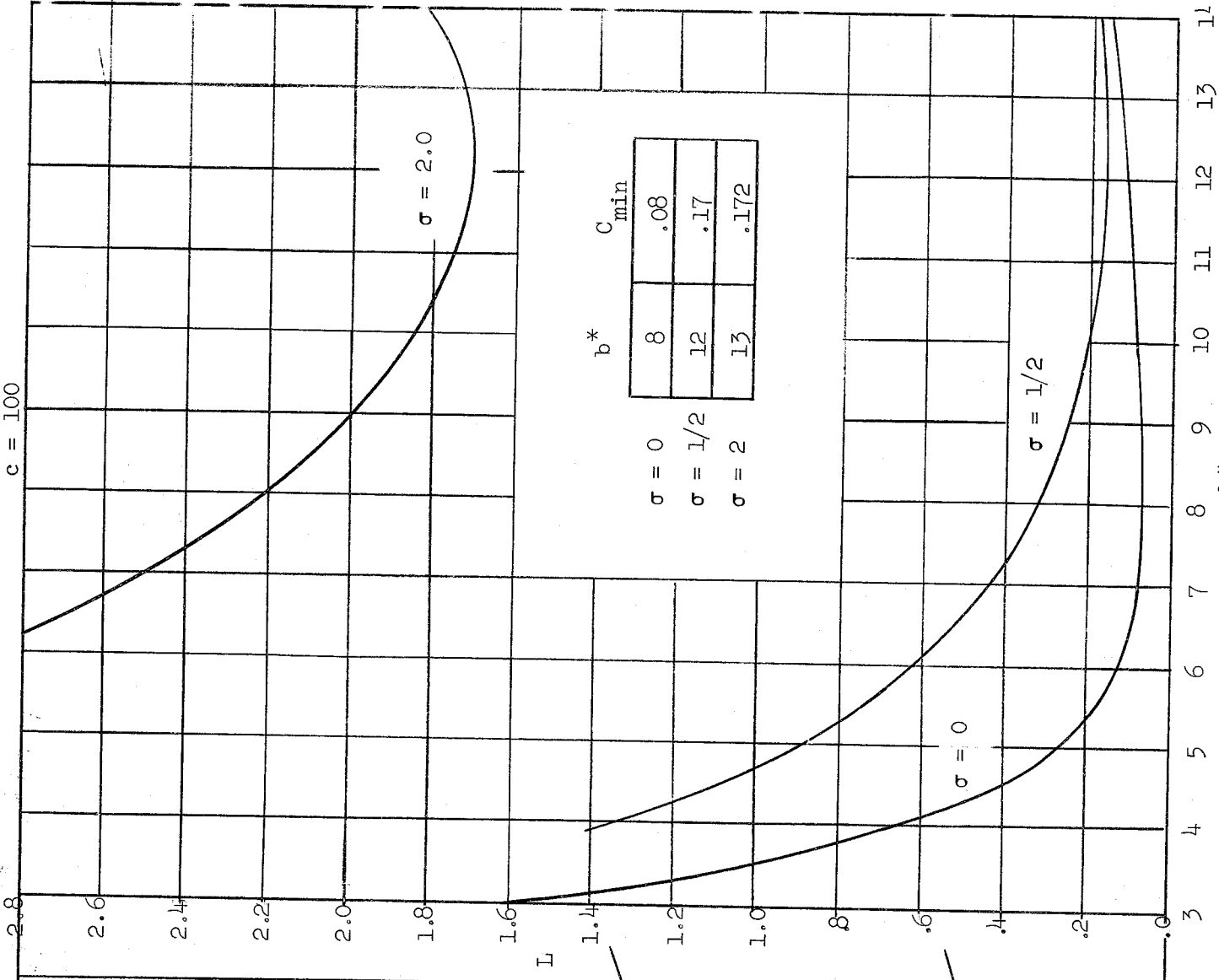
Figures

Average cost per transaction, L , as a function of reorder point

b^* for fixed values of p^* , s , c , and $\mu = 1$.

$p^* = .1$ $s = 20$

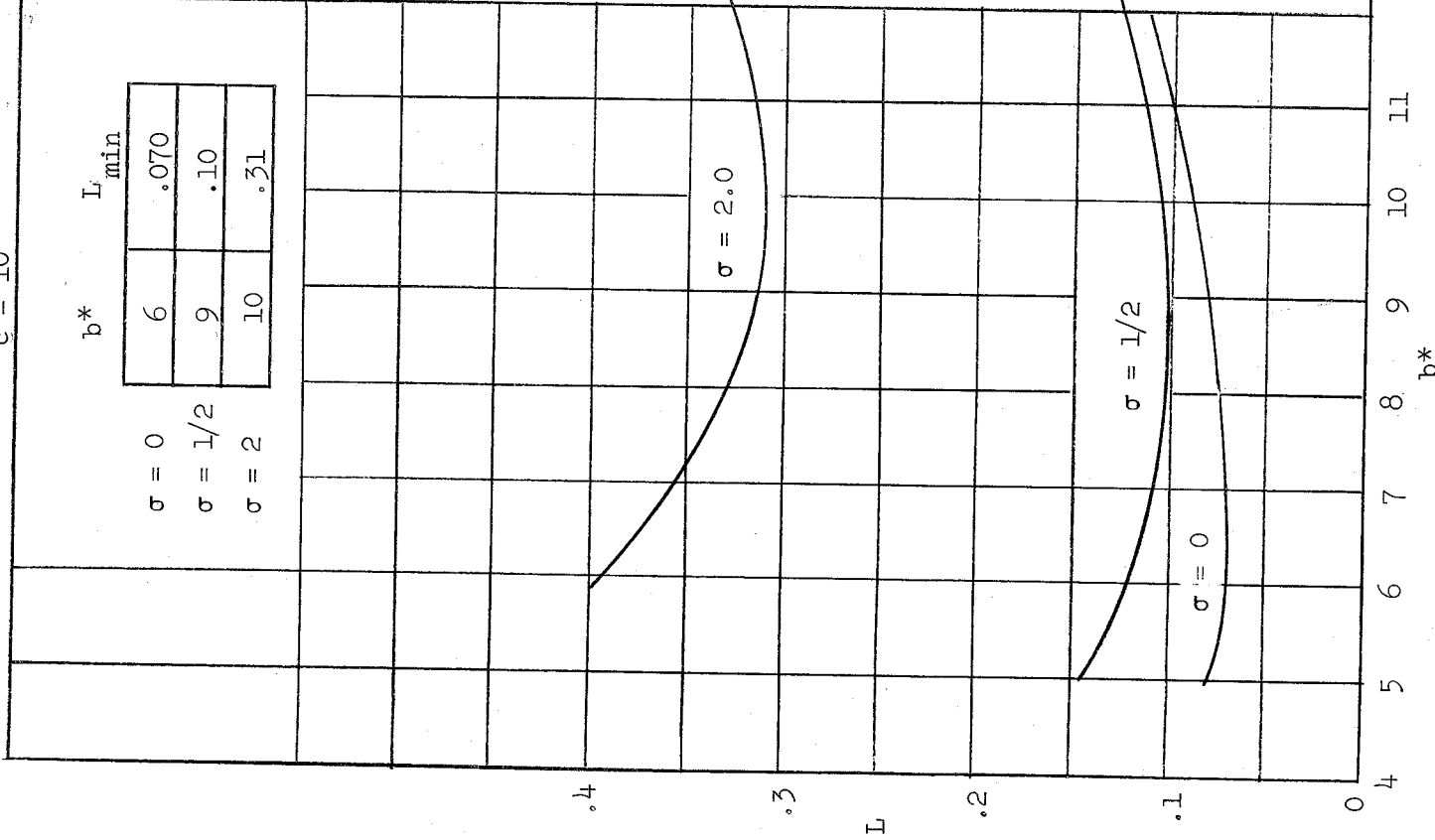
$c = 100$



$c = 10$

b^* L_{min}

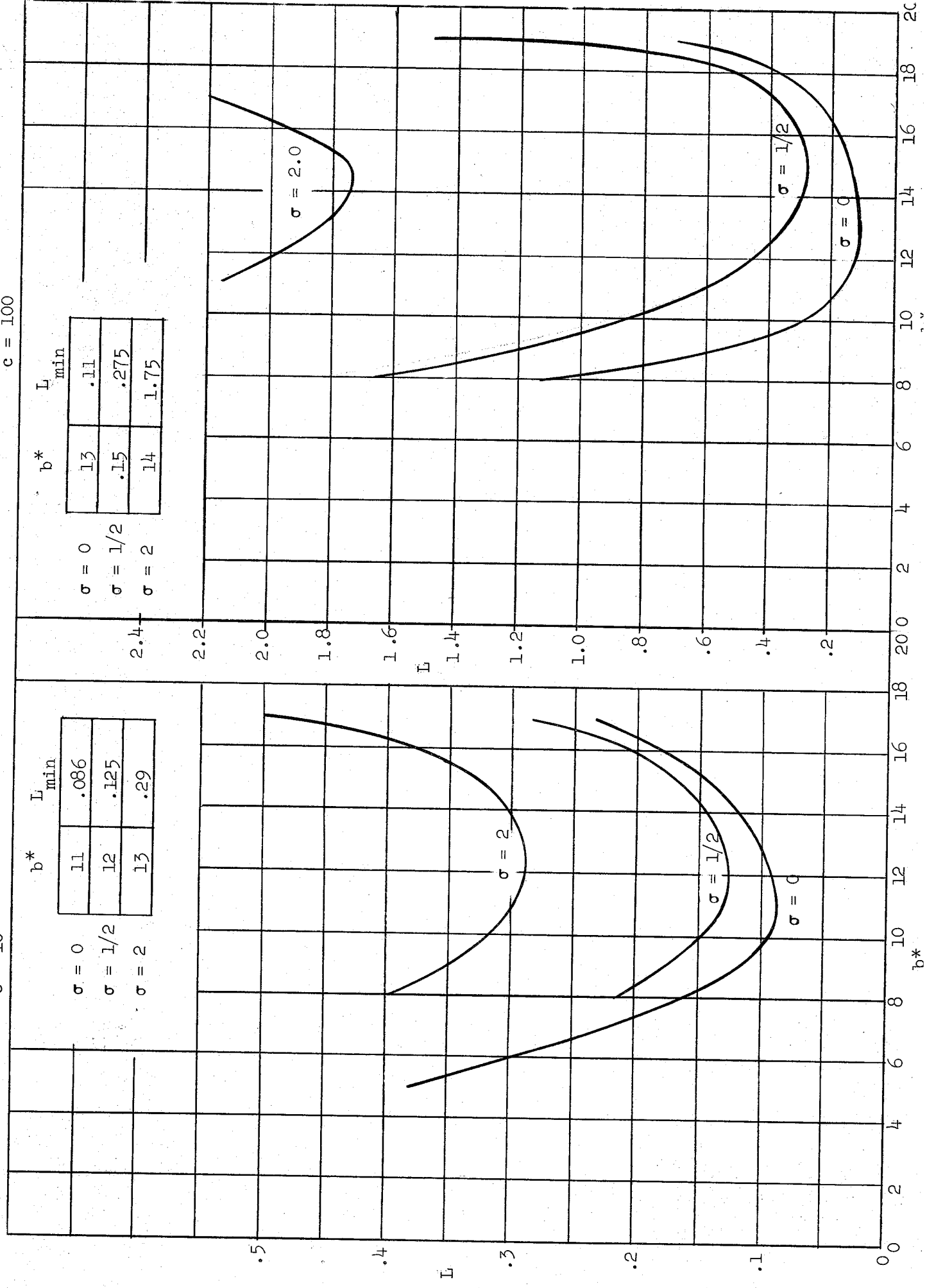
$\sigma = 0$	6	.070
$\sigma = 1/2$	9	.10
$\sigma = 2$	10	.31



$c = 10$ $p^* = .3$ $s = 20$ $c = 100$

σ	b^*	L_{min}
$\sigma = 0$	11	.086
$\sigma = 1/2$	12	.125
$\sigma = 2$	13	.29

σ	b^*	L_{min}
$\sigma = 0$	13	.11
$\sigma = 1/2$.15	.275
$\sigma = 2$	14	1.75

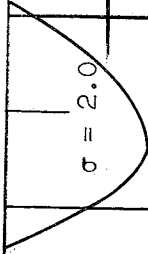


$p^* = .1$ $s = 100$

$c = 100$

b^*	L_{min}
23	.012
31	.013
53	.023

$\sigma = 0$
 $\sigma = 1/2$
 $\sigma = 2$

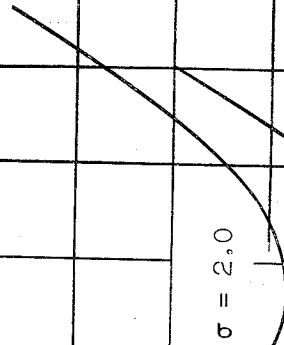


$p^* = .1$ $s = 100$

$c = 10$

b^*	L_{min}
20	.011
24	.012
37	.017

$\sigma = 0$
 $\sigma = 1/2$
 $\sigma = 2$



.024

.023

.022

.021

.020

.019

.018

.017

.016

.015

.014

.013

.012

.011

.010

L

10

20

30

40

50

b^*

20

30

40

50

60

.025

.024

.023

.022

.021

.020

.019

.018

.017

.016

.015

.014

.013

.012

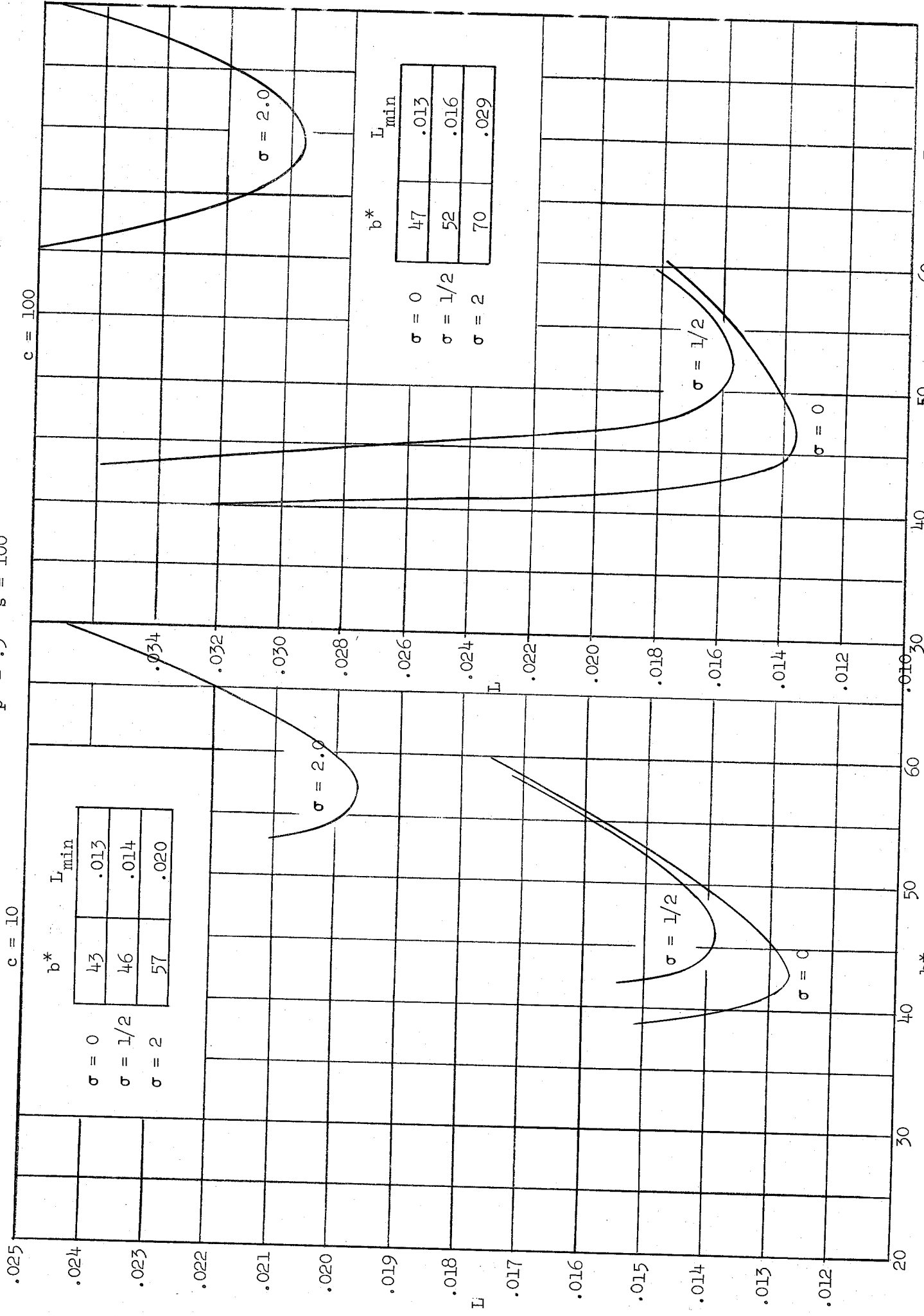
L

$\sigma = 0$

$\sigma = 1/2$

$\sigma = 2.0$

$p^* = .3 \quad s = 100$



$c = 10$

σ	b^*	L_{min}
0	43	.013
1/2	46	.014
2	57	.020

$c = 100$

σ	b^*	L_{min}
0	47	.013
1/2	52	.016
2	70	.029

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